Analyzing the Ramanujan's Integral, some equations concerning the Instantons and Topological Solitons. New possible mathematical connections with various parameters of Theoretical Physics and Supersymmetry Breaking. V

Michele Nardelli[1], **Antonio Nardelli**[2]

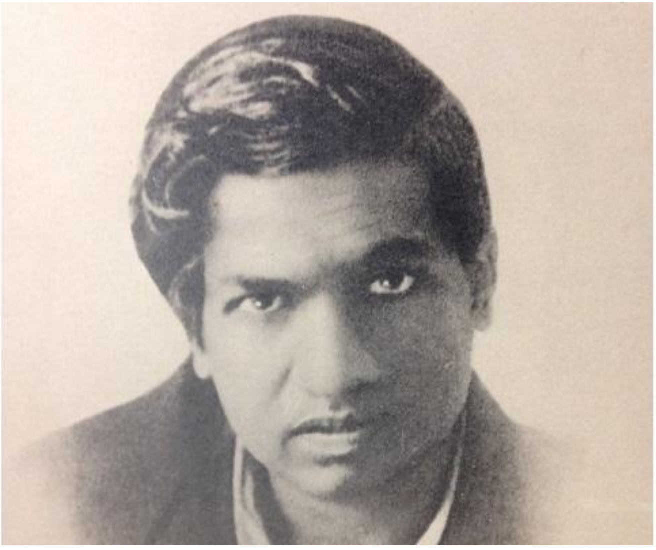
Abstract

In this research thesis (part V), we analyze the Ramanujan's Integral, some equations concerning the Instantons and Topological Solitons. We describe the new possible mathematical connections with various parameters of Theoretical Physics and Supersymmetry Breaking

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Vesuvius landscape with gorse – Naples

https://www.pinterest.it/pin/95068242114589901/

From:

A. A. Karatsuba, "On the zeros of arithmetic Dirichlet series without Euler product," Izv. Ross. Akad. Nauk, Ser. Mat. 57 (5), 3–14 (1993)

On the Zeros of the Davenport Heilbronn Function
S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

We have:

Let

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

and χ_1 be a character modulo 5 such that $\chi_1(2) = i$.

The Davenport–Heilbronn function $f(s)$ is defined by the equality

$$f(s) = \frac{1-i\varkappa}{2}L(s,\chi_1) + \frac{1+i\varkappa}{2}L(s,\overline{\chi}_1), \qquad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}.$$

The function $f(s)$ satisfies the Riemann-type functional equation

$$g(s) = g(1-s), \qquad \text{where} \quad g(s) = \left(\frac{\pi}{5}\right)^{-s/2}\Gamma\left(\frac{s+1}{2}\right)f(s),$$

but there is no Euler product for it.

$$(\sqrt{10-2\sqrt{5}}-2)/(\sqrt{5}-1) = \kappa$$

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

…

$0.28407904384\ldots = \kappa$

Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}} - 2\sqrt{5} + \sqrt{5\left(10-2\sqrt{5}\right)} - 2\right)$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}} - 2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5} + \sqrt{2\left(5+\sqrt{5}\right)}\right)$$

Minimal polynomial:

$$x^4 + 2x^3 - 6x^2 - 2x + 1$$

Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1} - \frac{2}{\sqrt{5}-1}$$

$$\frac{1}{4}\sqrt{10-2\sqrt{5}} + \frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)} + \frac{1}{2}\left(-1-\sqrt{5}\right)$$

For $((((\sqrt{(10-2\sqrt{5})} -2))/((\sqrt{5}-1)))) = 8\pi G$; $G = 0.011303146014$

Indeed:

(((($\sqrt{(10-2\sqrt{5})}$ -2))/(($\sqrt{5}$-1))))/(8π)

Input:

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

Result:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

Decimal approximation:

0.0113031460140052147973750129442035744685760313920017808594909667...

0.01130314…. = g (gravitational coupling constant)

Property:

$$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

Expanded forms:

$$-\frac{1}{16\pi}-\frac{\sqrt{5}}{16\pi}+\frac{\sqrt{10-2\sqrt{5}}}{32\pi}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi}-\frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\pi\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\pi\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

Input:

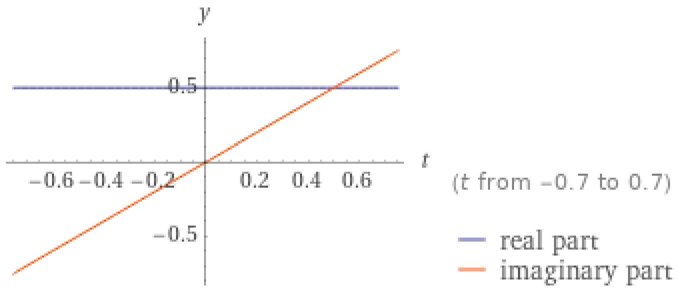
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1} \times \frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Plot:



Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}\;t}{\sqrt{2(5-\sqrt{5})}-2}-\frac{2\,i\,t}{\sqrt{2(5-\sqrt{5})}-2}+$$
$$\frac{\sqrt{5(10-2\sqrt{5})}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}-$$
$$\frac{\sqrt{5}}{(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}+\frac{1}{(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)}$$

Alternate forms:

$$\frac{1}{8}(1+\sqrt{5})\left(2\,i\sqrt{2(3-\sqrt{5})}\;t+\sqrt{5}-1\right)$$

$$\frac{1}{2}(1+2\,i\,t)$$

$$\frac{1}{2}+i\,t$$

1/2+*it* = *real part of every nontrivial zero of the Riemann zeta function*

Derivative:

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)(2\,i(\sqrt{5}-1)\,t+\sqrt{5}-1)}{(\sqrt{5}-1)\left(2\left(\sqrt{2(5-\sqrt{5})}-2\right)\right)}\right)=i$$

Indefinite integral:

$$\int \frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\,dt = \frac{t}{2}+\frac{i\,t^2}{2} + \text{constant}$$

And again:

(((√(10-2√5) -2)/((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2\,x}\times\frac{2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}=\frac{1}{2}+i\,t$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2\,i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are real:

$$\frac{\sqrt{5}-1}{x}=2$$

Alternate form:

$$\frac{\left(\sqrt{5}-1\right)(1+2\,i\,t)}{4\,x}=\frac{1}{2}+i\,t$$

Alternate form assuming t and x are positive:

$$2\,x+1=\sqrt{5}$$

Expanded forms:

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\;t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\;t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{i\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+i\,t$$

$$\frac{i\sqrt{5}\;t}{2\,x}-\frac{i\,t}{2\,x}+\frac{\sqrt{5}}{4\,x}-\frac{1}{4\,x}=\frac{1}{2}+i\,t$$

Solutions:

$$t=\frac{i}{2}\,,\quad x\neq 0$$

$$x=\frac{\sqrt{5}}{2}-\frac{1}{2}$$

Input:

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

Decimal approximation:

0.6180339887498948482045868343656381177203091798057628621354486227
…

0.6180339887…. $= \frac{1}{\phi}$

Solution for the variable x:

$$x = \frac{-2\,i\,\sqrt{5}\,t + 2\,i\,t - \sqrt{5} + 1}{-2 - 4\,i\,t}$$

Implicit derivatives:

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1 + \sqrt{5} - 2\,x\right)x}{\left(-1 + \sqrt{5}\right)(-i + 2\,t)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1 + \sqrt{5}\right)(-i + 2\,t)}{2\left(-1 + \sqrt{5} - 2\,x\right)x}$$

From:

Heterotic Supergravity with Internal Almost-Kähler Spaces; Instantons for SO(32), or E8 × E8, Gauge Groups; and Deformed Black Holes with Soliton, Quasiperiodic and/or Pattern-forming Structures - *Laurenţiu Bubuianu, Klee Irwin, Sergiu I. Vacaru* - arXiv:1611.00223v3 [physics.gen-ph] 18 Feb 2017

We have that:

$$ {}_{\shortmid}\overrightarrow{\mathbf{J}} := {}_{\shortmid}\overrightarrow{\mathbf{e}}^{5} \wedge {}_{\shortmid}\overrightarrow{\mathbf{e}}^{6} + {}_{\shortmid}\overrightarrow{\mathbf{e}}^{7} \wedge {}_{\shortmid}\overrightarrow{\mathbf{e}}^{8} + {}_{\shortmid}\overrightarrow{\mathbf{e}}^{9} \wedge {}_{\shortmid}\overrightarrow{\mathbf{e}}^{10} $$

$$ {}_{\shortmid}\overrightarrow{\Theta} := ({}_{\shortmid}\overrightarrow{\mathbf{e}}^{5} + i\,{}_{\shortmid}\overrightarrow{\mathbf{e}}^{6}) \wedge ({}_{\shortmid}\overrightarrow{\mathbf{e}}^{7} + i\,{}_{\shortmid}\overrightarrow{\mathbf{e}}^{8}) \wedge ({}_{\shortmid}\overrightarrow{\mathbf{e}}^{9} + {}_{\shortmid}\overrightarrow{\mathbf{e}}^{10}) $$

$$ \overrightarrow{*}_{z}\overrightarrow{\mathbf{F}} = -(\overrightarrow{*}_{z}\overrightarrow{\mathbf{Q}}_{z}) \wedge \overrightarrow{\mathbf{F}}, \text{ for } \overrightarrow{\mathbf{Q}}_{z} = d\tau \wedge {}_{\shortmid}^{+}\overrightarrow{\Theta} + \frac{1}{2}\,{}_{\shortmid}\overrightarrow{\mathbf{J}} \wedge {}_{\shortmid}\overrightarrow{\mathbf{J}}, \quad (69) $$

(e^5*e^6)+(e^7*e^8)+(e^9*e^10)

Input

$e^5\,e^6 + e^7\,e^8 + e^9\,e^{10}$

Exact result

$e^{11} + e^{15} + e^{19}$

Decimal approximation

$1.818111924773745693026672153206066829807397327886822013706 65\ldots \times 10^8$

1.818111924…*10^8

Property

$e^{11} + e^{15} + e^{19}$ is a transcendental number

Alternate forms

$$e^{11}\left(1+e^4+e^8\right)$$

$$e^{11}\left(1-e+e^2\right)\left(1+e+e^2\right)\left(1-e^2+e^4\right)$$

Alternative representation

$$e^5\,e^6+e^7\,e^8+e^9\,e^{10}=\exp^5(z)\exp^6(z)+\exp^7(z)\exp^8(z)+\exp^9(z)\exp^{10}(z)$$

for $z=1$

(e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)

Input

$$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)$$

i is the imaginary unit

Decimal approximation

$$-3.1330196995477568635282636007911134144205448880866786868 8\ldots\times 10^{10}+$$
$$2.665943258476098832343158526160279193374416165814656382 25\ldots\times 10^{10}\,i$$

Property

$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)$ is a transcendental number

Polar coordinates

$$r=e^{21}\left(1+e+e^2+e^3\right)\text{ (radius)},\quad \theta=\pi+\tan^{-1}\left(\frac{2\,e^{13}}{e^{12}-e^{14}}\right)\text{ (angle)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar forms

$$e^{21}\left(1+e+e^2+e^3\right)\left(\cos\left(\pi+\tan^{-1}\left(\frac{2\,e^{13}}{e^{12}-e^{14}}\right)\right)+i\sin\left(\pi+\tan^{-1}\left(\frac{2\,e^{13}}{e^{12}-e^{14}}\right)\right)\right)$$

Approximate form

$$e^{21}\left(1+e+e^2+e^3\right)e^{i\left(\pi+\tan^{-1}\left((2\,e^{13})/(e^{12}-e^{14})\right)\right)}$$

Alternate forms

$$-e^{21}\,(e+-i)^2\,(1+e)$$

$$-(e-1)\,e^{21}\,(1+e)^2+2\,i\,e^{22}\,(1+e)$$

Expanded form

$$e^{21}+(1+2\,i)\,e^{22}-(1-2\,i)\,e^{23}-e^{24}$$

Alternative representation

$$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)=\left(\exp^5(z)+i\exp^6(z)\right)\left(\exp^7(z)+i\exp^8(z)\right)\left(\exp^9(z)+\exp^{10}(z)\right)\ \text{for } z=1$$

Series representations

$$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left(\frac{5^{k_1}}{k_1!}+\frac{i\,6^{k_1}}{k_1!}\right)\left(\frac{7^{k_2}}{k_2!}+\frac{i\,8^{k_2}}{k_2!}\right)\left(\frac{9^{k_3}}{k_3!}+\frac{10^{k_3}}{k_3!}\right)$$

$$\left(e^{5}+i\,e^{6}\right)\left(e^{7}+i\,e^{8}\right)\left(e^{9}+e^{10}\right)=-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{21}\left(-i+\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2}\left(1+\sum_{k=0}^{\infty}\frac{1}{k!}\right)$$

$$\left(e^{5}+i\,e^{6}\right)\left(e^{7}+i\,e^{8}\right)\left(e^{9}+e^{10}\right)=\frac{\left(i+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{2}\left(1+\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)}{\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{24}}$$

(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2

Input

$$\left(e^{5}+i\,e^{6}\right)\left(e^{7}+i\,e^{8}\right)\left(e^{9}+e^{10}\right)+\frac{1}{2}\left(e^{5}\,e^{6}+e^{7}\,e^{8}+e^{9}\,e^{10}\right)^{2}$$

i is the imaginary unit

Exact result

$$\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^{2}+\left(e^{5}+i\,e^{6}\right)\left(e^{7}+i\,e^{8}\right)\left(e^{9}+e^{10}\right)$$

Decimal approximation

$$1.652762352482547611177913962257766729466879337603842391433\ldots\times 10^{16}+$$
$$2.665943258476098832343158526160279193374416165814656382257\ldots\times 10^{10}\,i$$

Property

$\left(e^{5}+i\,e^{6}\right)\left(e^{7}+i\,e^{8}\right)\left(e^{9}+e^{10}\right)+\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^{2}$ is a transcendental number

Polar coordinates

$$r = \sqrt{4\,e^{44}\,(1+e)^2 + \left(\frac{1}{2}\,e^{22}\,\left(1+e^4+e^8\right)^2 + e^9\,(1+e)\left(e^{12}-e^{14}\right)\right)^2}$$

(radius), $\theta = \tan^{-1}\left(\frac{2\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(e^{12}-e^{14}\right)+\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)$ (angle)

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 16527623524846978$ (radius), $\theta \approx 0$ (angle)

$1.6527623524846978 \times 10^{16}$

$\tan^{-1}(x)$ is the inverse tangent function

Polar forms

$$\sqrt{4\,e^{44}\,(1+e)^2 + \left(\frac{1}{2}\,e^{22}\,\left(1+e^4+e^8\right)^2 + e^9\,(1+e)\left(e^{12}-e^{14}\right)\right)^2}\left(\cos\left(\tan^{-1}\left(\frac{2\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(e^{12}-e^{14}\right)+\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right) + i\sin\left(\tan^{-1}\left(\frac{2\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(e^{12}-e^{14}\right)+\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)\right)$$

Approximate form

$$\sqrt{4\,e^{44}\,(1+e)^2 + \left(\frac{1}{2}\,e^{22}\,\left(1+e^4+e^8\right)^2 + e^9\,(1+e)\left(e^{12}-e^{14}\right)\right)^2}\exp\left(i\tan^{-1}\left(\frac{2\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(e^{12}-e^{14}\right)+\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)$$

Alternate forms

$$\frac{1}{2} e^{21} \left(2 + e \left((3 + 4 i) + e \left((-2 + 4 i) - 2 e + e^3 \left(1 + e^4\right) \left(2 + e^4 + e^8\right)\right)\right)\right)$$

$$\frac{1}{2} e^{21} \left(2 + (3 + 4 i) e - (2 - 4 i) e^2 - 2 e^3 + 2 e^5 + 3 e^9 + 2 e^{13} + e^{17}\right)$$

$$\frac{1}{2} e^{21} \left(2 + 3 e - 2 e^2 - 2 e^3 + 2 e^5 + 3 e^9 + 2 e^{13} + e^{17}\right) + 2 i e^{22} (1 + e)$$

Expanded form

$$e^{21} + \left(\frac{3}{2} + 2 i\right) e^{22} - (1 - 2 i) e^{23} - e^{24} + e^{26} + \frac{3 e^{30}}{2} + e^{34} + \frac{e^{38}}{2}$$

Alternative representation

$$\left(e^5 + i e^6\right)\left(e^7 + i e^8\right)\left(e^9 + e^{10}\right) + \frac{1}{2}\left(e^5 e^6 + e^7 e^8 + e^9 e^{10}\right)^2 = \left(\exp^5(z) + i \exp^6(z)\right)\left(\exp^7(z) + i \exp^8(z)\right)\left(\exp^9(z) + \exp^{10}(z)\right) + \frac{1}{2}\left(\exp^5(z) \exp^6(z) + \exp^7(z) \exp^8(z) + \exp^9(z) \exp^{10}(z)\right)^2 \quad \text{for } z = 1$$

Series representations

$$\left(e^5 + i e^6\right)\left(e^7 + i e^8\right)\left(e^9 + e^{10}\right) + \frac{1}{2}\left(e^5 e^6 + e^7 e^8 + e^9 e^{10}\right)^2 = \frac{1}{2}\left(\left(\sum_{k=0}^{\infty} \frac{11^k + 15^k + 19^k}{k!}\right)^2 + 2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \left(\frac{5^{k_1}}{k_1!} + \frac{i\, 6^{k_1}}{k_1!}\right)\left(\frac{7^{k_2}}{k_2!} + \frac{i\, 8^{k_2}}{k_2!}\right)\left(\frac{9^{k_3}}{k_3!} + \frac{10^{k_3}}{k_3!}\right)\right)$$

$$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2 = \frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{21}\left(2+(3+4\,i)\sum_{k=0}^{\infty}\frac{1}{k!}-(2-4\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^2 - 2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^3+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^5+3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^9+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{13}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{17}\right)$$

$$\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2 = \frac{1}{2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{38}} \left(1+2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^4+3\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^8+2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{12}-2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{14} - (2-4\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{15}+(3+4\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{16}+2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{17}\right)$$

z*y = -(x*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10))))))^2)*y

Input

$$z\,y = -\left(x\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)y$$

i is the imaginary unit

Exact result

$$y\,z = -\left(\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2+\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)x\right)y$$

Alternate forms

$$y\,z = -\frac{1}{2}\,e^{21}\left(e\left(1+e^4+e^8\right)^2-2\,(e+-i)^2\,(1+e)\,x\right)y$$

$$y\,z = -\frac{1}{2}\,e^{21}\left(-2\,e^{3}\,x-(2-4\,i)\,e^{2}\,x+(2+4\,i)\,e\,x+2\,x+e^{17}+2\,e^{13}+3\,e^{9}+2\,e^{5}+e\right)y$$

$$-e^{24}\,x\,y-(1-2\,i)\,e^{23}\,x\,y+(1+2\,i)\,e^{22}\,x\,y+e^{21}\,x\,y+y\,z+\frac{e^{38}\,y}{2}+e^{34}\,y+\frac{3\,e^{30}\,y}{2}+e^{26}\,y+\frac{e^{22}\,y}{2}=0$$

Expanded form

$$y\,z = e^{24}\,x\,y+(1-2\,i)\,e^{23}\,x\,y-(1+2\,i)\,e^{22}\,x\,y-e^{21}\,x\,y-\frac{e^{38}\,y}{2}-e^{34}\,y-\frac{3\,e^{30}\,y}{2}-e^{26}\,y-\frac{e^{22}\,y}{2}$$

Real solution

$$y=0$$

Solution

$$z \approx (31\,330\,196\,995-26\,659\,432\,585\,i)\,x-16\,527\,654\,855\,022\,471$$

Solution for the variable z

$$z=-\frac{1}{2}\,e^{21}\left(e\left(1+e^{4}+e^{8}\right)^{2}-2\,(e+-i)^{2}\,(1+e)\,x\right)$$

From:

$$z=-\frac{1}{2}\,e^{21}\left(e\left(1+e^{4}+e^{8}\right)^{2}-2\,(e+-i)^{2}\,(1+e)\,x\right)$$

((-1/2 e^21 (e (1 + e^4 + e^8)^2 - 2 (e + -i)^2 (1 + e) x)))

Input

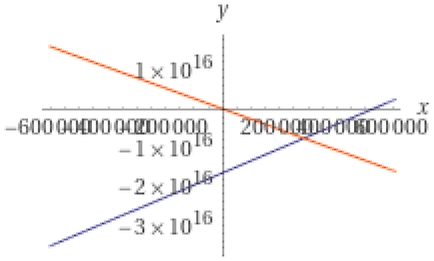
$$-\frac{1}{2} e^{21} \left(e \left(1+e^4+e^8\right)^2 - 2 (e-i)^2 (1+e)\, x\right)$$

i is the imaginary unit

Exact result

$$-\frac{1}{2} e^{21} \left(e \left(1+e^4+e^8\right)^2 - 2 (e+-i)^2 (1+e)\, x\right)$$

Plot



(x from −602647 to 602647)

Alternate forms

$$e^{21} (e+-i)^2 (1+e)\, x - \frac{1}{2} e^{22} \left(1-e+e^2\right)^2 \left(-1-i\, e+e^2\right)^2 \left(-1+i\, e+e^2\right)^2 \left(1+e+e^2\right)^2$$

$$\frac{1}{2} e^{21} \left(-e \left(1+e^4+e^8\right)^2 + 2 (e+-i)^2 (1+e)\, x\right)$$

$$-\frac{1}{2} e^{21} \left(-2 e^3 x - (2-4 i) e^2 x + (2+4 i) e\, x + 2 x + e^{17} + 2 e^{13} + 3 e^9 + 2 e^5 + e\right)$$

Expanded form

$$e^{24} x + (1-2 i) e^{23} x - (1+2 i) e^{22} x - e^{21} x - \frac{e^{38}}{2} - e^{34} - \frac{3 e^{30}}{2} - e^{26} - \frac{e^{22}}{2}$$

Alternate form assuming x is real

$$-\frac{1}{2} e^{21} \left(-2 e^3 x - 2 e^2 x + 2 e x + 2 x + e^{17} + 2 e^{13} + 3 e^9 + 2 e^5 + e\right) - 2 i e^{22} (1 + e)$$
$$x$$

Complex root

$$x = \frac{e \left(1 + e^4 + e^8\right)^2}{2 (e + -i)^2 (1 + e)}$$

Derivative

$$\frac{d}{dx}\left(-\frac{1}{2} e^{21} \left(e \left(1 + e^4 + e^8\right)^2 - 2 (e - i)^2 (1 + e) x\right)\right) = e^{21} (e + -i)^2 (1 + e)$$

Indefinite integral

$$\int -\frac{1}{2} e^{21} \left(e \left(1 + e^4 + e^8\right)^2 - 2 (e - i)^2 (1 + e) x\right) dx =$$
$$-\frac{1}{2} e^{21} \left(e \left(1 + e^4 + e^8\right)^2 x - (e + -i)^2 (1 + e) x^2\right) + \text{constant}$$

For x = 602647 :

((-1/2 e^21 (e (1 + e^4 + e^8)^2 - 2 (e + -i)^2 (1 + e) *602647)))

Input

$$-\frac{1}{2} e^{21} \left(e \left(1 + e^4 + e^8\right)^2 - 2 (e - i)^2 ((1 + e) \times 602647)\right)$$

i is the imaginary unit

Exact result

$$-\frac{1}{2}e^{21}\left(e\left(1+e^4+e^8\right)^2-1\,205\,294\,(e+-i)^2\,(1+e)\right)$$

Decimal approximation

$2.3533943737110987159993998370459460528000435702204217250 57\ldots \times 10^{15} -$
$1.6066227068908455330151074563149137750495117790797052247 98\ldots \times 10^{16}\, i$

Property

$-\frac{1}{2}e^{21}\left(-1\,205\,294\,(-i+e)^2\,(1+e)+e\left(1+e^4+e^8\right)^2\right)$ is a transcendental number

Polar coordinates

$r \approx 16\,237\,675\,859\,184\,461$ (radius), $\theta \approx -1.4253$ (angle)

16237675859184461 = z

$\tan^{-1}(x)$ is the inverse tangent function

Polar forms

$$\frac{1}{2}e^{21}\sqrt{5\,810\,934\,505\,744\,e^2\,(1+e)^2+\left(e\left(1+e^4+e^8\right)^2-1\,205\,294\,(1+e)\left(e^2-1\right)\right)^2}\left(\cos\left(-\tan^{-1}\left(\frac{2\,410\,588\,e\,(1+e)}{1\,205\,294\,(1+e)\left(e^2-1\right)-e\left(1+e^4+e^8\right)^2}\right)\right)+i\sin\left(-\tan^{-1}\left(\frac{2\,410\,588\,e\,(1+e)}{1\,205\,294\,(1+e)\left(e^2-1\right)-e\left(1+e^4+e^8\right)^2}\right)\right)\right)$$

Approximate form

$$\frac{1}{2} e^{21} \sqrt{5\,810\,934\,505\,744\, e^2 (1+e)^2 + \left(e\left(1+e^4+e^8\right)^2 - 1\,205\,294\,(1+e)\left(e^2-1\right)\right)^2} \exp\left(-i \tan^{-1}\left(\frac{2\,410\,588\, e\,(1+e)}{1\,205\,294\,(1+e)\left(e^2-1\right) - e\left(1+e^4+e^8\right)^2}\right)\right)$$

Alternate forms

$$-\frac{1}{2} e^{21} \left(1\,205\,294 + (1\,205\,295 + 2\,410\,588\, i)\, e - (1\,205\,294 - 2\,410\,588\, i)\, e^2 - 1\,205\,294\, e^3 + 2\, e^5 + 3\, e^9 + 2\, e^{13} + e^{17}\right)$$

$$-\frac{1}{2} e^{21} \left(1\,205\,294 + 1\,205\,295\, e - 1\,205\,294\, e^2 - 1\,205\,294\, e^3 + 2\, e^5 + 3\, e^9 + 2\, e^{13} + e^{17}\right) - 1\,205\,294\, i\, e^{22} (1+e)$$

$$-\frac{1}{2} e^{22} \left(1+e^4+e^8\right)^2 + 602\,647\, e^{21} (e + -i)^2 (1+e)$$

Expanded form

$$-602\,647\, e^{21} - \left(\frac{1\,205\,295}{2} + 1\,205\,294\, i\right) e^{22} + (602\,647 - 1\,205\,294\, i)\, e^{23} + 602\,647\, e^{24} - e^{26} - \frac{3\, e^{30}}{2} - e^{34} - \frac{e^{38}}{2}$$

Alternative representation

$$\frac{1}{2} \left(e^{21} \left(e\left(1+e^4+e^8\right)^2 - (1+e)\, 602\,647 \times 2\, (e-i)^2\right)\right)(-1) = \frac{1}{2} \left(\exp^{21}(z) \left(\exp(z)\left(1+\exp^4(z)+\exp^8(z)\right)^2 - (1+\exp(z))\, 602\,647 \times 2\, (\exp(z)-i)^2\right)\right)(-1) \quad \text{for } z = 1$$

Series representations

$$\frac{1}{2}\left(e^{21}\left(e\left(1+e^{4}+e^{8}\right)^{2}-(1+e)\,602\,647\times 2\,(e-i)^{2}\right)\right)(-1)=\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{21^{k}}{k!}\right)\left(-1\,205\,294-(1\,205\,295+2\,410\,588\,i)\,e+(1\,205\,294-2\,410\,588\,i)\,e^{2}+1\,205\,294\,e^{3}-2\,e\sum_{k=0}^{\infty}\frac{4^{k}+8^{k}}{k!}-e\left(\sum_{k=0}^{\infty}\frac{4^{k}+8^{k}}{k!}\right)^{2}\right)$$

$$\frac{1}{2}\left(e^{21}\left(e\left(1+e^{4}+e^{8}\right)^{2}-(1+e)\,602\,647\times 2\,(e-i)^{2}\right)\right)(-1)=-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{21}\left(1\,205\,294+(1\,205\,295+2\,410\,588\,i)\sum_{k=0}^{\infty}\frac{1}{k!}-(1\,205\,294-2\,410\,588\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2}-1\,205\,294\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{3}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{5}+3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{9}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{13}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{17}\right)$$

$$\frac{1}{2}\left(e^{21}\left(e\left(1+e^{4}+e^{8}\right)^{2}-(1+e)\,602\,647\times 2\,(e-i)^{2}\right)\right)(-1)=-\frac{1}{2\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{38}}\left(1+2\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{4}+3\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{8}+2\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{12}-1\,205\,294\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{14}-(1\,205\,294-2\,410\,588\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{15}+(1\,205\,295+2\,410\,588\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{16}+1\,205\,294\left(\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k!}\right)^{17}\right)$$

(16237675859184461)*y = -(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)*y

Input

$$16237675859184461\, y = -\left(602647\left(\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right) + \frac{1}{2}\left(e^5\, e^6 + e^7\, e^8 + e^9\, e^{10}\right)^2\right) y$$

For y = 1 :

-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)

Input

$$-\left(602647\left(\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right) + \frac{1}{2}\left(e^5\, e^6 + e^7\, e^8 + e^9\, e^{10}\right)^2\right)$$

i is the imaginary unit

Exact result

$$-\frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2 - 602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)$$

Decimal approximation

$$2.3533943737110987159993998370459460528000435702204217250 57\ldots \times 10^{15} - 1.6066227068908455330151074563149137750495117790797052247 98\ldots \times 10^{16}\, i$$

Property

$$-602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right) - \frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2$$
is a transcendental number

Polar coordinates

$$r = \sqrt{\left(1452733626436\, e^{44} (1+e)^2 + \left(602647\, e^{21} (1+e)(e^2-1) - \frac{1}{2} e^{22} (1+e^4+e^8)^2\right)^2\right)}$$

(radius),

$$\theta = -\tan^{-1}\left(\frac{1205294\, e^{13} (e^9+e^{10})}{(e^9+e^{10})(602647\, e^{14} - 602647\, e^{12}) - \frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right) \text{ (angle)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 16237675859184461$ (radius), $\theta \approx -1.4253$ (angle)

$1.6237675859184461 \times 10^{16}$

Polar forms

$$\sqrt{\left(1452733626436\, e^{44} (1+e)^2 + \left(602647\, e^{21} (1+e)(e^2-1) - \frac{1}{2} e^{22} (1+e^4+e^8)^2\right)^2\right)} \left(\cos\left(-\tan^{-1}\left(\frac{1205294\, e^{13} (e^9+e^{10})}{(e^9+e^{10})(602647\, e^{14} - 602647\, e^{12}) - \frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)\right) + i \sin\left(-\tan^{-1}\left(\frac{1205294\, e^{13} (e^9+e^{10})}{(e^9+e^{10})(602647\, e^{14} - 602647\, e^{12}) - \frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)\right)\right)$$

Approximate form

$$\sqrt{\left(1452733626436\, e^{44} (1+e)^2 + \left(602647\, e^{21} (1+e)(e^2-1) - \frac{1}{2} e^{22} (1+e^4+e^8)^2\right)^2\right)} \exp\left(-i \tan^{-1}\left(\frac{1205294\, e^{13} (e^9+e^{10})}{(e^9+e^{10})(602647\, e^{14} - 602647\, e^{12}) - \frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)\right)$$

Alternate forms

$$-\frac{1}{2} e^{22} \left(1+e^4+e^8\right)^2 + 602\,647\, e^{21} \,(e+-i)^2\,(1+e)$$

$$-\frac{1}{2} e^{21} \left(1\,205\,294 + (1\,205\,295 + 2\,410\,588\, i)\, e - (1\,205\,294 - 2\,410\,588\, i)\, e^2 - 1\,205\,294\, e^3 + 2\, e^5 + 3\, e^9 + 2\, e^{13} + e^{17}\right)$$

$$-\frac{1}{2} e^{21} \left(1\,205\,294 + 1\,205\,295\, e - 1\,205\,294\, e^2 - 1\,205\,294\, e^3 + 2\, e^5 + 3\, e^9 + 2\, e^{13} + e^{17}\right) - 1\,205\,294\, i\, e^{22}\,(1+e)$$

Expanded form

$$-602\,647\, e^{21} - \left(\frac{1\,205\,295}{2} + 1\,205\,294\, i\right) e^{22} + (602\,647 - 1\,205\,294\, i)\, e^{23} + 602\,647\, e^{24} - e^{26} - \frac{3\, e^{30}}{2} - e^{34} - \frac{e^{38}}{2}$$

Alternative representation

$$-\left(602\,647 \left(e^5 + i\, e^6\right)\left(\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right) + \frac{1}{2}\left(e^5\, e^6 + e^7\, e^8 + e^9\, e^{10}\right)^2\right) = -\left(602\,647 \left(\exp^5(z) + i \exp^6(z)\right)\left(\left(\exp^7(z) + i \exp^8(z)\right)\left(\exp^9(z) + \exp^{10}(z)\right)\right) + \frac{1}{2}\left(\exp^5(z)\exp^6(z) + \exp^7(z)\exp^8(z) + \exp^9(z)\exp^{10}(z)\right)^2\right) \text{ for } z = 1$$

Series representations

$$-\left(602\,647 \left(e^5 + i\, e^6\right)\left(\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right) + \frac{1}{2}\left(e^5\, e^6 + e^7\, e^8 + e^9\, e^{10}\right)^2\right) = \frac{1}{2}\left(-\left(\sum_{k=0}^{\infty} \frac{11^k + 15^k + 19^k}{k!}\right)^2 - 1\,205\,294 \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty} \left(\frac{5^{k_1}}{k_1!} + \frac{i\, 6^{k_1}}{k_1!}\right)\left(\frac{7^{k_2}}{k_2!} + \frac{i\, 8^{k_2}}{k_2!}\right)\left(\frac{9^{k_3}}{k_3!} + \frac{10^{k_3}}{k_3!}\right)\right)$$

$$-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{21}\left(1205294+(1205295+2410588\,i)\sum_{k=0}^{\infty}\frac{1}{k!}-\right.$$
$$(1205294-2410588\,i)\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^2-1205294\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^3+$$
$$\left.2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^5+3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^9+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{13}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{17}\right)$$

$$-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$-\frac{1}{2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{38}}\left(1+2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^4+3\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^8+2\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{12}-\right.$$
$$1205294\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{14}-(1205294-2410588\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{15}+$$
$$\left.(1205295+2410588\,i)\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{16}+1205294\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{17}\right)$$

From which:

(((1+1/(64*6)) + π/log(2))) ((-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)))

Input

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)$$
$$\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\left(\frac{385}{384}+\frac{\pi}{\log(2)}\right)$$

Decimal approximation

$1.3025953862301193362830712467523759732374585392044578717 29\ldots \times 10^{16} -$
$8.8925993398566231324302856214846140137585465346095492033 31\ldots \times 10^{16}\, i$

Polar coordinates

$$r=\frac{1}{384\log(2)}\sqrt{\left(1452733626436\,e^{44}(1+e)^2+\left(602647\,e^{21}(1+e)\left(e^2-1\right)-\frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)}\,(384\pi+385\log(2))\ \text{(radius)},$$

$$\theta=-\tan^{-1}\left(\frac{1205294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)$$

(angle)

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 89874956333478373$ (radius), $\theta \approx -1.4253$ (angle)

$8.9874956333478373 \cdot 10^{16} \approx c^2$, where c is the value of speed of light 299792458

Polar forms

$$\frac{1}{384\log(2)}\sqrt{\left(1452733626436\,e^{44}(1+e)^2+\left(602647\,e^{21}(1+e)\left(e^2-1\right)-\frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)}\,(384\pi+385\log(2))$$
$$\left(\cos\left(-\tan^{-1}\left(\frac{1205294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)+\right.$$
$$\left.i\sin\left(-\tan^{-1}\left(\frac{1205294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)\right)$$

Approximate form

$$\frac{1}{384\log(2)}\sqrt{\left(1\,452\,733\,626\,436\,e^{44}\,(1+e)^2+\left(602\,647\,e^{21}\,(1+e)\left(e^2-1\right)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)}\,(384\,\pi+385\log(2))\exp\left(-i\tan^{-1}\left(\frac{1\,205\,294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\,e^{14}-602\,647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)$$

Alternate forms

$$\left(-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2+602\,647\,e^{21}\,(e+-i)^2\,(1+e)\right)\left(\frac{385}{384}+\frac{\pi}{\log(2)}\right)$$

$$-\frac{1}{768\log(2)}e^{21}\left(1\,205\,294+(1\,205\,295+2\,410\,588\,i)\,e-(1\,205\,294-2\,410\,588\,i)\,e^2-1\,205\,294\,e^3+2\,e^5+3\,e^9+2\,e^{13}+e^{17}\right)(384\,\pi+385\log(2))$$

$$-\frac{1}{768\log(2)}e^{21}\left(1\,205\,294+1\,205\,295\,e-1\,205\,294\,e^2-1\,205\,294\,e^3+2\,e^5+3\,e^9+2\,e^{13}+e^{17}\right)(384\,\pi+385\log(2))-\frac{602\,647\,i\,e^{22}\,(1+e)\,(384\,\pi+385\log(2))}{192\log(2)}$$

$$-\frac{232\,019\,095}{384}\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{385}{768}\left(e^{11}+e^{15}+e^{19}\right)^2-\frac{602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\pi}{\log(2)}-\frac{\left(e^{11}+e^{15}+e^{19}\right)^2\pi}{2\log(2)}$$

Expanded form

$$-\frac{232019095\,e^{21}}{384}-\left(\frac{154679525}{256}+\frac{232019095\,i}{192}\right)e^{22}+\left(\frac{232019095}{384}-\frac{232019095\,i}{192}\right)e^{23}+\frac{232019095\,e^{24}}{384}-\frac{385\,e^{26}}{384}-\frac{385\,e^{30}}{256}-\frac{385\,e^{34}}{384}-\frac{385\,e^{38}}{768}-\frac{602647\,e^{21}\,\pi}{\log(2)}-\frac{\left(\frac{1205295}{2}+1205294\,i\right)e^{22}\,\pi}{\log(2)}+\frac{(602647-1205294\,i)\,e^{23}\,\pi}{\log(2)}+\frac{602647\,e^{24}\,\pi}{\log(2)}-\frac{e^{26}\,\pi}{\log(2)}-\frac{3\,e^{30}\,\pi}{2\log(2)}-\frac{e^{34}\,\pi}{\log(2)}-\frac{e^{38}\,\pi}{2\log(2)}$$

Alternative representations

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=\left(1+\frac{1}{384}+\frac{\pi}{\log_e(2)}\right)\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)$$

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=\left(1+\frac{1}{384}+\frac{\pi}{\log(a)\log_a(2)}\right)\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)$$

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)$$
$$\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(1+\frac{1}{384}+\frac{\pi}{2\coth^{-1}(3)}\right)$$
$$\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)$$

Series representations

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)$$
$$\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)$$
$$\left(\frac{385}{384}+\frac{\pi}{2\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}}\right)\quad \text{for } x<0$$

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)$$
$$\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)$$
$$\left(\frac{385}{384}+\frac{\pi}{\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}}\right)$$

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\left(\frac{385}{384}+\frac{\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}}\right)$$

Integral representations

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(602\,647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)\left(\frac{385}{384}+\frac{\pi}{\int_1^2\frac{1}{t}\,dt}\right)$$

$$\left(\left(1+\frac{1}{64\times 6}\right)+\frac{\pi}{\log(2)}\right)(-1)\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)=$$
$$\left(602\,647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)\left(\frac{385}{384}+\frac{2\,i\,\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\quad\text{for } -1<\gamma<0$$

We obtain also:

61/ ln((-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)))

Input

$$\frac{61}{\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\frac{61}{\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)}$$

Decimal approximation

1.6318651541220494773412324565793783464071699367737669655896564… + 0.062315058945628667525245253726002638643092475437581414147788194… i

Polar coordinates

$r \approx 1.6331$ (radius), $\theta = 0.0381679$ (angle)

1.6331 result very near to the mean between $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... and the value of golden ratio 1.61803398…, i.e. 1.63148399

$\tan^{-1}(x)$ is the inverse tangent function

Polar forms

$$(122\,(\cos(0.0381679) + i\,\sin(0.0381679))) \Big/ \left(\sqrt{\left(\log^2\left(1\,452\,733\,626\,436\,e^{44}\,(1+e)^2 + \left(602\,647\,e^{21}\,(1+e)\left(e^2-1\right) - \frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^2\right) + 4\tan^{-1}\left(\frac{1\,205\,294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\,e^{14} - 602\,647\,e^{12}\right) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)^2\right)}\right)$$

Approximate form

$$\left(122\,e^{0.0381679\,i}\right) \Big/ \left(\sqrt{\left(\log^2\left(1\,452\,733\,626\,436\,e^{44}\,(1+e)^2 + \left(602\,647\,e^{21}\,(1+e)\left(e^2-1\right) - \frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^2\right) + 4\tan^{-1}\left(\frac{1\,205\,294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\,e^{14} - 602\,647\,e^{12}\right) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)^2\right)}\right)$$

Alternate forms

$$\frac{61}{\log\left(-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2 + 602\,647\,e^{21}\,(e+-i)^2\,(1+e)\right)}$$

$$61 \Big/ \left(21 + \log\left(\frac{1}{2}\left(-1\,205\,294 - (1\,205\,295 + 2\,410\,588\,i)\,e + (1\,205\,294 - 2\,410\,588\,i)\,e^2 + 1\,205\,294\,e^3 - 2\,e^5 - 3\,e^9 - 2\,e^{13} - e^{17}\right)\right)\right)$$

$$61\bigg/\left(\frac{1}{2}\log\left(1\,452\,733\,626\,436\,e^{26}\left(e^9+e^{10}\right)^2+\left(\left(e^9+e^{10}\right)\left(602\,647\,e^{14}-602\,647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)^2\right)-i\tan^{-1}\left(\frac{1\,205\,294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\,e^{14}-602\,647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)$$

Alternative representations

$$\frac{61}{\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=\frac{61}{\log_e\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)}$$

$$\frac{61}{\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=\frac{61}{\log(a)\log_a\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)}$$

$$\frac{61}{\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=-\frac{61}{\mathrm{Li}_1\left(1+602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)}$$

Series representations

$$\frac{61}{\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=61\bigg/\left(\log\left(-1-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+602\,647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^{-k}}{k}\right)$$

$$\frac{61}{\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=$$

$$61\Bigg/\left(2\,i\,\pi\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-x\right)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-x\right)^k x^{-k}}{k}\right)$$

for $x < 0$

$$\frac{61}{\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=$$

$$61\Bigg/\left(\log(z_0)+\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-z_0\right)\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-z_0\right)^k z_0^{-k}}{k}\right)$$

Integral representations

$$\frac{61}{\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)}=\frac{61}{\int_1^{602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2}\frac{1}{t}\,dt}$$

$$\frac{61}{\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)} = \frac{122\,i\,\pi}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\left(-1-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \text{ for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

48 ((ln((-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)))-2^(1/3)))-Pi

Input

$$48\left(\log\left(-\left(602\,647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$-\pi+48\left(-\sqrt[3]{2}+\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)$$

Decimal approximation

1728.035506101065123612660912371533223092685840423614204254409848… − 68.41679368406827463228359490991562545161610283429631532957502659… i

Input interpretation

$1728.0355061 + i\times(-68.4167936)$

i is the imaginary unit

Result

$1728.035506\ldots -$
$68.4167936\ldots\, i$

Polar coordinates

$r = 1729.39$ (radius), $\theta = -0.0395716$ (angle)

1729.39

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Polar forms

$$\sqrt{\left(\left(48\sqrt[3]{2} + \pi - 24\log\left(1\,452\,733\,626\,436\, e^{44}\,(1+e)^2 + \left(602\,647\, e^{21}\,(1+e)\left(e^2-1\right) - \frac{1}{2}\, e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)\right)^2 + 2304\tan^{-1}\left(\frac{1\,205\,294\, e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\, e^{14} - 602\,647\, e^{12}\right) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)^2\right)}$$

$$\left(\cos\left(\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right) \Big/ \left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)\right)\right) + i\sin\left(\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right) \Big/ \left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)\right)\right)\right)$$

Approximate form

$$\sqrt{\left(\left(48\sqrt[3]{2}+\pi-24\log\left(1452733626436\,e^{44}\,(1+e)^2+\left(602647\,e^{21}\,(1+e)\left(e^2-1\right)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)\right)^2+2304\tan^{-1}\left(\frac{1205294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\Big/\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)\right)\right)$$

Alternate forms

$$-\pi+48\left(-\sqrt[3]{2}+\log\left(-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2+602647\,e^{21}\,(e+-i)^2\,(1+e)\right)\right)$$

$$1008-48\sqrt[3]{2}-\pi-48\log(2)+48\log\left(-1205294-(1205295+2410588\,i)\,e+(1205294-2410588\,i)\,e^2+1205294\,e^3-2\,e^5-3\,e^9-2\,e^{13}-e^{17}\right)$$

$$-\pi+48\left(-\sqrt[3]{2}+\frac{1}{2}\log\left(1452733626436\,e^{26}\left(e^9+e^{10}\right)^2+\left(\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)^2\right)-i\tan^{-1}\left(\frac{1205294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602647\,e^{14}-602647\,e^{12}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)$$

Alternative representations

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-\pi+48\left(\log_e\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)$$

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-\pi+48\left(\log(a)\log_a\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)$$

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-\pi+48\left(-\text{Li}_1\left(1+602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)$$

Series representations

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-48\sqrt[3]{2}-\pi+48\log\left(-1-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)-48\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^{-k}}{k}$$

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-48\sqrt[3]{2}-\pi+96\,i\,\pi\left\lfloor\frac{\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-x\right)}{2\pi}\right\rfloor+48\log(x)-48\sum_{k=1}^{\infty}\frac{(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-48\sqrt[3]{2}-\pi+96\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+48\log(z_0)-48\sum_{k=1}^{\infty}\frac{(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-z_0\right)^k z_0^{-k}}{k}$$

Integral representations

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-48\sqrt[3]{2}-\pi+48\int_1^{602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2}\frac{1}{t}\,dt$$

$$48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi=-48\sqrt[3]{2}-\pi-\frac{24\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1+602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds\quad\text{for } -1<\gamma<0$$

((1/27 ((48 ((ln((-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)))-2^(1/3)))-Pi))))^2+276-(e*ln8)

Input

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left((e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)$$

log(x) is the natural logarithm

i is the imaginary unit

Exact result

$$276-e\log(8)+\frac{1}{729}\left(-\pi+48\left(-\sqrt[3]{2}+\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)^2$$

Decimal approximation

4360.094889221077448034368494699547668901688711860436910089462857… − 324.3529456780825152362343605535824812417921457693459436518793850… i

Polar coordinates

$r = 4372.142755718505672972680308179953065576447178505130325724238569$ (radius), $\theta = -0.0742544837479408504073738585468182177557603020193343740830916554$ (angle)

4372.1427557185…. ≈ 4372

where 4372 is a value indicated in the fundamental Ramanujan paper "**Modular equations and Approximations to π**"

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Polar forms

$$
\sqrt{\begin{aligned}&\Bigg(\Bigg(276 - e\log(8) + \frac{1}{729}\Big(\Big(48\sqrt[3]{2} + \pi - 24\log\Big(1\,452\,733\,626\,436\,e^{44}(1+e)^2 + \\
&\qquad \Big(602\,647\,e^{21}(1+e)(e^2-1) - \\
&\qquad\qquad \frac{1}{2}e^{22}(1+e^4+e^8)^2\Big)^2\Big)\Big)^2 - \\
&\quad 2304\tan^{-1}\Big(\big(1\,205\,294\,e^{13}(e^9+e^{10})\big)\Big/ \\
&\qquad \Big((e^9+e^{10})(602\,647\,e^{14}-602\,647\,e^{12}) - \\
&\qquad\qquad \frac{1}{2}(e^{11}+e^{15}+e^{19})^2\Big)\Big)^2\Big)\Big)^2 + \\
&\quad \frac{1}{59\,049}1024\Big(48\sqrt[3]{2}+\pi-24\log\Big(1\,452\,733\,626\,436\,e^{44}(1+e)^2 + \\
&\qquad \Big(602\,647\,e^{21}(1+e)(e^2-1) - \frac{1}{2}e^{22}(1+e^4+e^8)^2\Big)^2\Big)\Big)^2 \\
&\quad \tan^{-1}\left(\frac{1\,205\,294\,e^{13}(e^9+e^{10})}{(e^9+e^{10})(602\,647\,e^{14}-602\,647\,e^{12})-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)^2\Bigg)\end{aligned}}
$$

$$
\begin{aligned}
&\Bigg(\cos\Bigg(\tan^{-1}\Bigg(\Bigg(32\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - \\
&\qquad 602\,647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\Big)\Big) \\
&\quad \Big(-\pi+48\Big(-\sqrt[3]{2}+\mathrm{Re}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - \\
&\qquad 602\,647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\Big)\Big)\Big)\Big)\Big)\Big/ \\
&\quad \Big(243\Big(\frac{1}{729}\Big(\Big(-\pi+48\Big(-\sqrt[3]{2}+\mathrm{Re}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602\,647 \\
&\qquad (e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\Big)\Big)\Big)\Big)^2 - \\
&\qquad 2304\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602\,647 \\
&\qquad (e^5+i\,e^6)(e^7+i\,e^8) \\
&\qquad (e^9+e^{10})\Big)\Big)^2\Big) + 276 - e\log(8)\Big)\Big)\Big)\Big) + \\
&\quad i\sin\Bigg(\tan^{-1}\Bigg(\Bigg(32\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602\,647(e^5+i\,e^6) \\
&\qquad (e^7+i\,e^8)(e^9+e^{10})\Big)\Big) \\
&\quad \Big(-\pi+48\Big(-\sqrt[3]{2}+\mathrm{Re}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602\,647 \\
&\qquad (e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\Big)\Big)\Big)\Big)\Big)\Big/\Big(243 \\
&\quad \Big(\frac{1}{729}\Big(\Big(-\pi+48\Big(-\sqrt[3]{2}+\mathrm{Re}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602\,647 \\
&\qquad (e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\Big)\Big)\Big)\Big)^2 - \\
&\qquad 2304\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - \\
&\qquad 602\,647(e^5+i\,e^6)(e^7+i\,e^8) \\
&\qquad (e^9+e^{10})\Big)\Big)^2\Big) + 276 - e\log(8)\Big)\Big)\Big)\Big)\Bigg)
\end{aligned}
$$

Approximate form

$$\sqrt{\Bigg(\Bigg(276 - e\log(8) + \frac{1}{729}\Bigg(\Bigg(48\sqrt[3]{2} + \pi - 24\log\Big(1\,452\,733\,626\,436\,e^{44}(1+e)^2 + \Big(602\,647\,e^{21}(1+e)(e^2-1) - \frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\Big)^2\Big)\Bigg)^2 - 2304\tan^{-1}\Big(\left(1\,205\,294\,e^{13}(e^9+e^{10})\right)\Big/\Big((e^9+e^{10})(602\,647\,e^{14} - 602\,647\,e^{12}) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\Big)\Big)^2\Bigg)^2 + \frac{1}{59\,049}1024\Bigg(48\sqrt[3]{2} + \pi - 24\log\Big(1\,452\,733\,626\,436\,e^{44}(1+e)^2 + \Big(602\,647\,e^{21}(1+e)(e^2-1) - \frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\Big)^2\Big)\Bigg)^2 \tan^{-1}\left(\frac{1\,205\,294\,e^{13}(e^9+e^{10})}{(e^9+e^{10})(602\,647\,e^{14}-602\,647\,e^{12}) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)^2\Bigg) \exp\Bigg(i\tan^{-1}\Bigg(\Bigg(32\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647(e^5+ie^6)(e^7+ie^8)(e^9+e^{10})\Big)\Big)\Big(-\pi + 48\Big(-\sqrt[3]{2} + \mathrm{Re}\Big(\log\Big(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647(e^5+ie^6)(e^7+ie^8)(e^9+e^{10})\Big)\Big)\Big)\Big)\Bigg)\Bigg/\Bigg(243\Bigg(\frac{1}{729}\Bigg(\Big(-\pi + 48\Big(-\sqrt[3]{2} + \mathrm{Re}\Big(\log\Big(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647(e^5+ie^6)(e^7+ie^8)(e^9+e^{10})\Big)\Big)\Big)\Big)^2 - 2304\,\mathrm{Im}\Big(\log\Big(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647(e^5+ie^6)(e^7+ie^8)(e^9+e^{10})\Big)\Big)^2\Bigg) + 276 - e\log(8)\Bigg)\Bigg)\Bigg)\Bigg)}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\mathrm{Im}(z)$ is the imaginary part of z

$\mathrm{Re}(z)$ is the real part of z

Alternate forms

$$276 - e\log(8) + \frac{1}{729}\left(\pi + 48\left(\sqrt[3]{2} - \log\left(-\frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2 + 602\,647\,e^{21}\,(e + -i)^2\,(1+e)\right)\right)\right)^2$$

$$276 - e\log(8) + \frac{1}{729}\left(-\pi + 48\left(-\sqrt[3]{2} + \frac{1}{2}\log\left(1\,452\,733\,626\,436\,e^{26}\left(e^9+e^{10}\right)^2 + \left(\left(e^9+e^{10}\right)\left(602\,647\,e^{14} - 602\,647\,e^{12}\right) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)^2\right) - i\tan^{-1}\left(\frac{1\,205\,294\,e^{13}\left(e^9+e^{10}\right)}{\left(e^9+e^{10}\right)\left(602\,647\,e^{14} - 602\,647\,e^{12}\right) - \frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2}\right)\right)\right)^2$$

$$\frac{1}{729}\Bigg(201\,204 + 2304\times 2^{2/3} + 96\sqrt[3]{2}\,\pi + \pi^2 + 2304\log^2\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right) - 729\,e\log(8) - 4608\sqrt[3]{2}\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right) - 96\,\pi\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\Bigg)$$

Expanded form

$$276 + \frac{256\times 2^{2/3}}{81} + \frac{32\sqrt[3]{2}\,\pi}{243} + \frac{\pi^2}{729} + \frac{256}{81}\log^2\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right) - e\log(8) - \frac{512}{81}\sqrt[3]{2}\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right) - \frac{32}{243}\pi\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2 - 602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)$$

Alternative representations

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)=276-e\log_e(8)+\left(\frac{1}{27}\left(-\pi+48\left(\log_e\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\right)^2$$

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)=276-e\log(a)\log_a(8)+\left(\frac{1}{27}\left(-\pi+48\left(\log(a)\log_a\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\right)^2$$

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)=276+e\,\mathrm{Li}_1(-7)+\left(\frac{1}{27}\left(-\pi+48\left(-\mathrm{Li}_1\left(1+602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\right)^2$$

Series representations

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)=276-e\left(\log(7)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{7}\right)^k}{k}\right)+\frac{1}{729}\left(-\pi+48\left(-\sqrt[3]{2}+\log\left(-1-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(-1+602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^{-k}\right)\right)^2$$

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+276-e\log(8)=276-e\left(2\,i\,\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(8-x)^k\,x^{-k}}{k}\right)+\frac{1}{729}\left(-\pi+48\left(-\sqrt[3]{2}+2\,i\,\pi\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-x\right)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-x\right)^k\,x^{-k}\right)\right)^2\quad\text{for } x<0$$

$$\left(\frac{1}{27}\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\right)^2+$$

$$276-e\log(8)=276-e\left(\log(z_0)+\left\lfloor\frac{\arg(8-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(8-z_0)^k\,z_0^{-k}}{k}\right)+$$

$$\frac{1}{729}\left(-\pi+48\left(-\sqrt[3]{2}+\log(z_0)+\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-z_0\right)\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-z_0\right)^k\,z_0^{-k}\right)\right)^2$$

(((48 ((ln((-(602647*(((e^5+i*e^6)(e^7+i*e^8)(e^9+e^10)))+1/2(((((e^5*e^6)+(e^7*e^8)+(e^9*e^10)))))^2)))-2^(1/3)))-Pi)))^1/15

Input

$$\left(48\left(\log\left(-\left(602647\left(\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\verb|^|(1/15)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\left(-\pi+48\left(-\sqrt[3]{2}+\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\verb|^|(1/15)$$

Decimal approximation

1.643834184563778556862358132056453640334814123948719897475196... − 0.004336616723596455228539986136884467122685794617301816507649631... i

Input interpretation

$1.643834184 + i \times (-0.004336616)$

i is the imaginary unit

Result

1.643834184... −
0.004336616 i

Polar coordinates

$r = 1.64384$ (radius), $\theta = -0.0026381$ (angle)

$1.64384 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... (trace of the instanton shape)

Polar forms

$$\left(\left(48\sqrt[3]{2} + \pi - 24\log\left(1452733626436\, e^{44}(1+e)^2 + \left(602647\, e^{21}(1+e)(e^2-1) - \frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)\right)^2 + 2304 \tan^{-1}\left(\frac{1205294\, e^{13}(e^9+e^{10})}{(e^9+e^{10})(602647\, e^{14} - 602647\, e^{12}) - \frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)^2\right)\hat{}$$

$$(1/30)\left(\cos\left(\frac{1}{15}\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\Big/\left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\right)\right)\right) + i\sin\left(\frac{1}{15}\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\Big/\left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2 - 602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\right)\right)\right)\right)$$

Approximate form

$$\left(\left(48\sqrt[3]{2}+\pi-24\log\left(1452733626436\,e^{44}(1+e)^2+\left(602647\,e^{21}(1+e)\right.\right.\right.\right.$$
$$\left.\left.\left.(e^2-1)-\frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2\right)^2\right)\right)^2+2304$$
$$\left.\tan^{-1}\left(\frac{1205294\,e^{13}(e^9+e^{10})}{(e^9+e^{10})(602647\,e^{14}-602647\,e^{12})-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2}\right)^2\right)\hat{}$$
$$(1/30)\exp\left(\frac{1}{15}\,i\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2-\right.\right.\right.\right.\right.$$
$$\left.\left.\left.602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\Big/$$
$$\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-\frac{1}{2}(e^{11}+e^{15}+e^{19})^2-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\left.602647(e^5+i\,e^6)(e^7+i\,e^8)(e^9+e^{10})\right)\right)\right)\right)\right)\right)\right)$$

Alternate forms

$$\sqrt[15]{-\pi+48\left(-\sqrt[3]{2}+\log\left(-\frac{1}{2}e^{22}\left(1+e^4+e^8\right)^2+602647\,e^{21}(e+-i)^2(1+e)\right)\right)}$$

$$\left(-\pi+48\left(21-\sqrt[3]{2}+\log\left(\right.\right.\right.$$
$$\frac{1}{2}\left(-1205294-(1205295+2410588\,i)\,e+(1205294-2410588\,i)\right.$$
$$\left.\left.\left.\left.e^2+1205294\,e^3-2\,e^5-3\,e^9-2\,e^{13}-e^{17}\right)\right)\right)\right)\hat{}\,(1/15)$$

$$\left(-\pi + 48\left(-\sqrt[3]{2} + \frac{1}{2}\log\left(1452733626436\, e^{26}\left(e^9 + e^{10}\right)^2 + \left(\left(e^9 + e^{10}\right)\left(602647\, e^{14} - 602647\, e^{12}\right) - \frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2\right)^2\right) - i\tan^{-1}\left(\frac{1205294\, e^{13}\left(e^9 + e^{10}\right)}{\left(e^9 + e^{10}\right)\left(602647\, e^{14} - 602647\, e^{12}\right) - \frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2}\right)\right)\right)\text{^}(1/15)$$

All 15th roots of -π + 48 (-2^(1/3) + log(-1/2 (e^11 + e^15 + e^19)^2 - 602647 (e^5 + i e^6) (e^7 + i e^8) (e^9 + e^10)))

$$\left(2304\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2 - 602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right)\right)^2 + \left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2 - 602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right)\right)\right)\right)\right)^2\right)\text{^}(1/30)$$
$$\exp\left(\frac{1}{15}\, i\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right) - \frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2\right)\right)\right) \Big/ \left(-\pi + 48\left(-\sqrt[3]{2} + \mathrm{Re}\left(\log\left(-602647\left(e^5 + i\, e^6\right)\left(e^7 + i\, e^8\right)\left(e^9 + e^{10}\right) - \frac{1}{2}\left(e^{11} + e^{15} + e^{19}\right)^2\right)\right)\right)\right)\right)\right) \approx 1.64383 - 0.00434\, i$$

$$\left(2304\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)^2+\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)^2\right)\hat{}\,(1/30)\exp\left(\frac{1}{15}\,i\left(2\pi+\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\Big/\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\right)\right)\right)\right)\approx 1.50348+0.6646\,i \text{ (principal root)}$$

$$\left(2304\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)^2+\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)^2\right)\hat{}\,(1/30)\exp\left(\frac{1}{15}\,i\left(4\pi+\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\Big/\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\right)\right)\right)\right)\approx 1.1032+1.2187\,i$$

$$\left(2304\,\mathrm{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)^2+\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)^2\right)\hat{}\,(1/30)\exp\left(\frac{1}{15}\,i\left(6\pi+\tan^{-1}\left(\left(48\,\mathrm{Im}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\Big/\left(-\pi+48\left(-\sqrt[3]{2}+\mathrm{Re}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\right)\right)\right)\right)\approx 0.5121+1.56204\,i$$

$$\left(2304\,\text{Im}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)^2+\left(-\pi+48\left(-\sqrt[3]{2}+\text{Re}\left(\log\left(-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)\right)\right)\right)^2\right)\text{^}(1/30)$$

$$\exp\left(\frac{1}{15}\,i\left(8\,\pi+\tan^{-1}\left(\left(48\,\text{Im}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\Big/\left(-\pi+48\left(-\sqrt[3]{2}+\text{Re}\left(\log\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)\right)\right)\right)\right)\right)\right)\approx-0.16751+1.63528\,i$$

Alternative representations

$$\left(48\left(\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\text{^}(1/15)=\left(-\pi+48\left(\log_e\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\text{^}(1/15)$$

$$\left(48\left(\log\left(-\left(602\,647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\text{^}(1/15)=\left(-\pi+48\left(\log(a)\log_a\left(-602\,647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\text{^}(1/15)$$

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\text{^}(1/15)=$$

$$\left(-\pi+48\left(-\text{Li}_1\left(1+602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)-\sqrt[3]{2}\right)\right)\text{^}(1/15)$$

Series representations

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\text{^}(1/15)=$$

$$\left(-\pi+48\left(-\sqrt[3]{2}+\log\left(-1-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2\right)^{-k}}{k}\right)\right)\text{^}(1/15)$$

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\text{^}(1/15)=$$

$$\left(-\pi+48\left(-\sqrt[3]{2}+2\,i\,\pi\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-x\right)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-x\right)^k x^{-k}\right)\right)\text{^}(1/15)\quad\text{for } x<0$$

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\wedge(1/15)=$$

$$\left(-\pi+48\left(-\sqrt[3]{2}+\log(z_0)+\left\lfloor\frac{1}{2\pi}\arg\left(-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2-z_0\right)\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2-z_0\right)^k\,z_0^{-k}}{k}\right)\right)\wedge(1/15)$$

Integral representations

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\wedge(1/15)=$$

$$\sqrt[15]{-48\sqrt[3]{2}-\pi+48\int_1^{602647\,e^{21}\,(-i+e)^2\,(1+e)-\frac{1}{2}\,e^{22}\left(1+e^4+e^8\right)^2}\frac{1}{t}\,dt}$$

$$\left(48\left(\log\left(-\left(602647\left(e^5+i\,e^6\right)\left(\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)\right)+\frac{1}{2}\left(e^5\,e^6+e^7\,e^8+e^9\,e^{10}\right)^2\right)\right)-\sqrt[3]{2}\right)-\pi\right)\wedge(1/15)=$$

$$\left(-\pi+48\left(-\sqrt[3]{2}-\frac{i}{2\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{1}{\Gamma(1-s)}\left(-1-602647\left(e^5+i\,e^6\right)\left(e^7+i\,e^8\right)\left(e^9+e^{10}\right)-\frac{1}{2}\left(e^{11}+e^{15}+e^{19}\right)^2\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)\,ds\right)\right)\wedge(1/15)\quad\text{for } -1<\gamma<0$$

From:

ZETA-FUNCTION REGULARIZATION AND MULTI-INSTANTON DETERMINANTS - *E. CORRIGAN, P. GODDARD, H. OSBORN, S. TEMPLETON* - Received 25 June 1979

Now, we have that:

$$\kappa^2 = \det\begin{pmatrix} \alpha & \beta \\ \gamma & \chi \end{pmatrix} = |\alpha\gamma^{-1}\chi\gamma - \beta\gamma|^2 \ , \qquad (4.2)$$

$$\delta\zeta'(0) = \frac{1}{8\pi^2}\int \operatorname{tr} a_2(x, x)\, \delta\omega(x)\, d^4x \qquad (4.14)$$

$$= \frac{1}{96\pi^2}\int \operatorname{tr}(F_{\mu\nu}F_{\mu\nu})\, \delta\omega(x)\, d^4x \qquad (4.15)$$

$$\tfrac{1}{2}\operatorname{tr}(F_{\mu\nu}F_{\mu\nu}) = \partial^2 T \ ,$$

$$\frac{1}{96\pi^2}\int \operatorname{tr}(F_{\mu\nu}F_{\mu\nu})\, \delta\omega\, d^4x = \frac{1}{48\pi^2}\int \partial^2 T(\sigma + \tau_\mu \chi_\mu)\, d^4x \qquad (4.24)$$

$$= \frac{1}{48\pi^2}\lim_{r\to\infty}\int \{\partial_\nu T(\sigma + \tau_\mu x_\mu) - T\tau_\nu\}\, dS_\nu \ , \qquad (4.25)$$

$$\delta\zeta'(0) = \tfrac{1}{24}\operatorname{tr}_W[(b^+a\,\tau + \tau^+a^+b)(b^+b)^{-1}] - \tfrac{1}{6}k\sigma \qquad (4.27)$$

$$= \tfrac{1}{12}\operatorname{tr}_W[(b^+a\,\delta\gamma + \delta\gamma^+a^+b)(b^+b)^{-1}] - \tfrac{1}{12}k\operatorname{tr}(\delta\chi - \delta\alpha) \qquad (4.28)$$

$$= \tfrac{1}{12}\operatorname{tr}_W[\delta(b^+b)(b^+b)^{-1}] - \tfrac{1}{12}k\operatorname{tr}(\delta\chi + \delta\alpha) \qquad (4.29)$$

$$= \tfrac{1}{12}\,\delta\ln[\det(b^+b)^2\kappa^{-2k}] \ , \qquad (4.30)$$

For k = 2 and (((($\sqrt{}$(10-2$\sqrt{}$5) -2))/(($\sqrt{}$5-1)))) = κ:

1/12 δ ln[((k^2)^2*(((($\sqrt{}$(10-2$\sqrt{}$5) -2))/(($\sqrt{}$5-1))))^(-4))]

Input

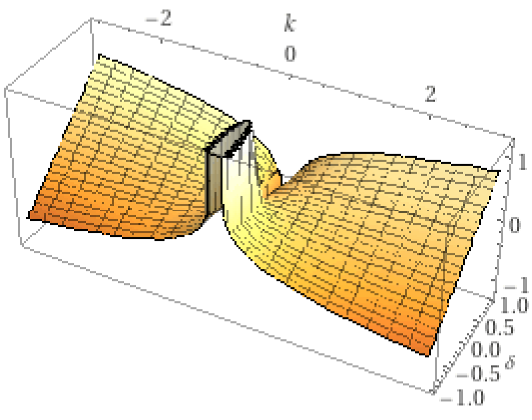
$$\frac{1}{12}\,\delta\log\left(\frac{(k^2)^2}{\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^4}\right)$$

log(x) is the natural logarithm

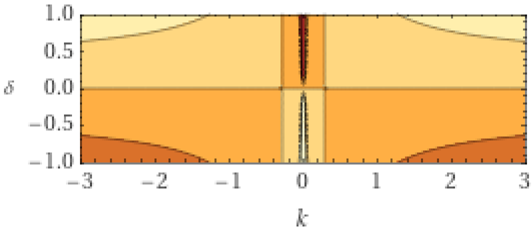
Exact result

$$\frac{1}{12}\,\delta\log\left(\frac{(\sqrt{5}-1)^4 k^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)$$

3D plot **(figures that can be related to the D-branes/Instantons)**



Contour plot



Alternate forms

$$\frac{1}{12}\,\delta\log\left(\left(41+16\sqrt{5}+4\sqrt{185+82\sqrt{5}}\right)k^4\right)$$

$$\frac{1}{12}\,\delta\left(\log(k^4)-4\log\left(\sqrt{10-2\sqrt{5}}-2\right)+4\log\left(\sqrt{5}-1\right)\right)$$

Alternate forms assuming k and δ are positive

$$\frac{1}{3}\,\delta\log\left(\frac{1}{2}\left(1+\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)k\right)$$

$$\frac{1}{3}\,\delta\left(\log(k)-\log\left(\sqrt{10-2\sqrt{5}}-2\right)+\log\left(\sqrt{5}-1\right)\right)$$

$$-\frac{1}{3}\,\delta\log\left(\sqrt{10-2\sqrt{5}}-2\right)+\frac{1}{3}\,\delta\log\left(\sqrt{5}-1\right)+\frac{1}{3}\,\delta\log(k)$$

Alternate form assuming k and δ are real

$$\frac{1}{3}\,\delta\log\left(\sqrt{5}-1\right)+\frac{1}{12}\,\delta\log\left(\frac{k^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)$$

Real roots

$$k = \frac{2 - \sqrt{10 - 2\sqrt{5}}}{\sqrt{5} - 1}$$

$$k = \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

$$\delta = 0$$

Complex roots

$$k \approx -0.28408\, i$$

$$k \approx 0.28408\, i$$

Property as a function

Parity

odd

Series expansion at k=0

$$\frac{1}{12}\,\delta \log\left(\left(41 + 16\sqrt{5} + 4\sqrt{185 + 82\sqrt{5}}\right)k^4\right) + O\left(k^6\right)$$

(generalized Puiseux series)

Series expansion at k=∞

$$\frac{1}{12}\,\delta \log\left(\left(41 + 16\sqrt{5} + 4\sqrt{185 + 82\sqrt{5}}\right)k^4\right) + O\left(\left(\frac{1}{k}\right)^6\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial k}\left(\frac{1}{12}\,\delta \log\left(\frac{(k^2)^2}{\left(\frac{\sqrt{10-2\sqrt{5}}\;-2}{\sqrt{5}\;-1}\right)^4}\right)\right) = \frac{\delta}{3\,k}$$

Indefinite integral

$$\int \frac{1}{12}\,\delta \log\left(\frac{\left(-1+\sqrt{5}\right)^4 k^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right) dk =$$

$$\frac{1}{12}\,\delta\, k\left(\log\left(\frac{(\sqrt{5}\;-1)^4 k^4}{\left(\sqrt{10-2\sqrt{5}}\;-2\right)^4}\right)-4\right) + \text{constant}$$

For k = 2 and δ = 0.5 , we obtain:

1/12 *0.5* log(((sqrt(5) - 1)^4 * 2^4)/(sqrt(10 - 2 sqrt(5)) - 2)^4)

Input

$$\frac{1}{12}\times 0.5 \log\left(\frac{(\sqrt{5}\;-1)^4\times 2^4}{\left(\sqrt{10-2\sqrt{5}}\;-2\right)^4}\right)$$

$\log(x)$ is the natural logarithm

Result

0.3252749894539850203670708941508681587574025638210254399438368774
…

0.32527498945….

Alternative representations

$$\frac{1}{12} \times 0.5 \log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = \frac{1}{12} \times 0.5 \log_e\left(\frac{2^4\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

$$\frac{1}{12} \times 0.5 \log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = \frac{1}{12} \times 0.5 \log(a) \log_a\left(\frac{2^4\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

$$\frac{1}{12} \times 0.5 \log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = \frac{1}{12}\,(-0.5)\,\text{Li}_1\left(1-\frac{2^4\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

Series representations

$$\frac{1}{12} \times 0.5 \log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = 0.0416667 \log\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right) - 0.0416667 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-k}}{k}$$

$$\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$0.0833333\,i\,\pi\left\lfloor\frac{\arg\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)}{2\pi}\right\rfloor+0.0416667\log(x)-$$

$$0.0416667\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^k}{k}\quad \text{for } x<0$$

$$\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$0.0416667\log\left(\frac{16\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4}{\left(-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)^4}\right)$$

Integral representations

$$\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=0.0416667\int_1^{\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}}\frac{1}{t}\,dt$$

$$\frac{1}{12} \times 0.5 \log\left(\frac{(\sqrt{5}-1)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) =$$

$$\frac{0.0208333}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left(-1+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-s}}{\Gamma(1-s)} \, ds \quad \text{for } -1<\gamma<0$$

From which:

1+2(((1/12 *0.5* log(((sqrt(5) - 1)^4 * 2^4)/(sqrt(10 - 2 sqrt(5)) - 2)^4))))

Input

$$1+2\left(\frac{1}{12} \times 0.5 \log\left(\frac{(\sqrt{5}-1)^4 \times 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)$$

$\log(x)$ is the natural logarithm

Result

1.650549978907970040734141788301736317514805127642050879887673754 8...

1.6505499789…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Alternative representations

$$1+\frac{2}{12}\left(0.5 \log\left(\frac{(\sqrt{5}-1)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right) = 1+\frac{1}{12} \log_e\left(\frac{2^4 (-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

$$1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=1+\frac{1}{12}\log(a)\log_a\left(\frac{2^4\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

$$1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=1-\frac{1}{12}\operatorname{Li}_1\left(1-\frac{2^4\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)$$

Series representations

$$1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=$$
$$1+0.0833333\log\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)-$$
$$0.0833333\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-k}}{k}$$

$$1+\frac{2}{12}\left(0.5\log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=$$

$$1+0.166667\,i\,\pi\left\lfloor\frac{\arg\left(-x+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)}{2\pi}\right\rfloor+0.0833333\log(x)-$$

$$0.0833333\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^k}{k}\quad\text{for } x<0$$

$$1+\frac{2}{12}\left(0.5\log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=$$

$$0.0833333\left(12+\log\left(\frac{16\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4}{\left(-2+\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)^4}\right)\right)$$

Integral representations

$$1+\frac{2}{12}\left(0.5\log\left(\frac{\left(\sqrt{5}-1\right)^4 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=1+0.0833333\int_1^{\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}}\frac{1}{t}\,dt$$

$$1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)=$$

$$1+\frac{0.0416667}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-s}}{\Gamma(1-s)}\,ds \text{ for } -1<\gamma<0$$

(((1+2(((1/12 *0.5* log(((sqrt(5) - 1)^4 * 2^4)/(sqrt(10 - 2 sqrt(5)) - 2)^4)))))))^15-76-29-4-(((1/12 *0.5* log(((sqrt(5) - 1)^4 * 2^4)/(sqrt(10 - 2 sqrt(5)) - 2)^4))))

Input

$$\left(1+2\left(\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\times 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15}-$$
$$76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\times 2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)$$

$\log(x)$ is the natural logarithm

Result

1729.03…

1729.03….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15} - 76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = -109 - \frac{1}{12}\times 0.5\log_e\left(\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right) + \left(1+\frac{1}{12}\log_e\left(\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)\right)^{15}$$

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15} - 76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right) = -109 - \frac{1}{12}\times 0.5\log(a)\log_a\left(\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right) + \left(1+\frac{1}{12}\log(a)\log_a\left(\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)\right)^{15}$$

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15}-$$

$$76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$-109+\frac{1}{12}\times 0.5\,\mathrm{Li}_1\left(1-\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)+$$

$$\left(1-\frac{1}{12}\,\mathrm{Li}_1\left(1-\frac{2^4\,(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)\right)^{15}$$

Series representations

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15}-$$

$$76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$-109+\left(1+0.0833333\left(\log\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-k}}{k}\right)\right)^{15}-0.0416667$$

$$\left(\log\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{16\left(-1+\sqrt{5}\right)^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^{-k}}{k}\right)$$

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15}-$$

$$76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$-109+\left(1+0.0833333\log\left(\frac{16\left(-1+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4}{\left(-2+\sqrt{9-2\sqrt{5}}\,\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)^4}\right)\right)^{15}-$$

$$0.0416667\log\left(\frac{16\left(-1+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4}{\left(-2+\sqrt{9-2\sqrt{5}}\,\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2\sqrt{5})^{-k}\right)^4}\right)$$

$$\left(1+\frac{2}{12}\left(0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)\right)\right)^{15}-$$

$$76-29-4-\frac{1}{12}\times 0.5\log\left(\frac{(\sqrt{5}-1)^4\,2^4}{\left(\sqrt{10-2\sqrt{5}}-2\right)^4}\right)=$$

$$-109+\left(1+0.0833333\left(2\,i\,\pi\left\lfloor\frac{\arg\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^k}{k}\right)\right)^{15}-$$

$$0.0416667\left(2\,i\,\pi\left\lfloor\frac{\arg\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{16(-1+\sqrt{5})^4}{\left(-2+\sqrt{10-2\sqrt{5}}\right)^4}\right)^k}{k}\right)\quad \text{for } x<0$$

$\binom{n}{m}$ is the binomial coefficient

From:

Lectures on the Mass of Topological Solitons - Heat kernel/Zeta function control of one-loop divergences - *A. Alonso Izquierdo, W. Garcia Fuertes, M.A. Gonzalez Leon M. de la Torre Mayado, J. Mateos Guilarte, J.M. Munoz Castaneda* - arXiv:hep-th/0611180v2 - 5 Sep 2007

We have that:

If σ is a natural number, the reflection scattering coefficient is zero for both K^{11} and K^{22}. The Cahill-Comtet-Glauber (CCG) formula can be applied. This formula gives the one-loop mass shift of one-dimensional solitons from the energies of their bound states. Applied to the TK1 kink of the BNRT model it reads:

$$\Delta M(\vec{\phi}^{TK1}) = -\frac{\hbar m}{\pi}\left(\sum_{i=0}^{1} 2(\sin\theta_i - \theta_i\cos\theta_i) + \sum_{l=0}^{N-1} N(\sin\alpha_l - \alpha_l\cos\alpha_l)\right) \quad . \qquad (11)$$

The angles are defined in terms of the eigenvalues of the bound states of K^{11} and K^{22}:

$$\theta_0 = \arccos(\frac{0}{2}) = \frac{\pi}{2} \quad , \quad \theta_1 = \arccos(\frac{\sqrt{3}}{2}) = \frac{\pi}{6} \quad , \quad \alpha_l = \arccos(\frac{\sqrt{(2N-l)l}}{N}) \quad .$$

Thus, for the first five cases we obtain:

1. $\sigma = 1$

$$\alpha_0 = \frac{\pi}{2} \quad , \quad \sin\theta_0 = \sin\alpha_0 = 1 \quad , \quad \sin\theta_1 = \frac{1}{2}$$

$$\Delta M(\vec{\phi}^{TK1}) = (-\frac{3}{\pi} + \frac{1}{2\sqrt{3}})\hbar m - \frac{1}{\pi}\hbar m = -0.984564\hbar m \quad .$$

2. $\sigma = 2$

$$\alpha_1 = \frac{\pi}{6} \quad , \quad \Delta M(\vec{\phi}^{TK1}) = (-\frac{3}{\pi} + \frac{1}{2\sqrt{3}})\hbar m - (\frac{3}{\pi} - \frac{1}{2\sqrt{3}})\hbar m = -1.33251\hbar m \quad .$$

3. $\sigma = 3$

$$\alpha_1 = \arccos(\frac{\sqrt{5}}{3}) \quad , \quad \sin\alpha_1 = \frac{2}{3} \quad , \quad \alpha_2 = \arccos(2\frac{\sqrt{2}}{3}) \quad , \quad \sin\alpha_2 = \frac{1}{3}$$

$$\Delta M(\vec{\phi}^{TK1}) = (-\frac{3}{\pi}+\frac{1}{2\sqrt{3}})\hbar m - (\frac{6}{\pi} - \frac{1}{\pi}(\sqrt{5}\arccos(\frac{\sqrt{5}}{3}) + 2\sqrt{2}\arccos(2\frac{\sqrt{2}}{3})))\hbar m = -1.75076\hbar m \quad .$$

4. $\sigma = 4$

$$\alpha_1 = \arccos(\frac{\sqrt{7}}{4}) \quad , \quad \sin\alpha_1 = \frac{3}{4} \quad , \quad \alpha_2 = \arccos(2\frac{\sqrt{3}}{4}) \quad , \quad \sin\alpha_2 = \frac{2}{4}$$

$$\alpha_3 = \arccos(\frac{\sqrt{15}}{4}) \quad , \quad \sin\alpha_3 = \frac{1}{4}$$

$$\begin{aligned} \Delta M(\vec{\phi}^{TK1}) &= (-\frac{3}{\pi} + \frac{1}{2\sqrt{3}})\hbar m - (\frac{10}{\pi} - \frac{1}{\pi}(\sqrt{7}\arccos(\frac{\sqrt{7}}{4}) + 2\sqrt{3}\arccos(\frac{2\sqrt{3}}{4}) + \\ &+ \sqrt{15}\arccos(\frac{\sqrt{15}}{4})))\hbar m = -2.24628\hbar m \quad . \end{aligned}$$

5. $\sigma = 5$

$$\alpha_1 = \arccos(\frac{\sqrt{9}}{5}) \quad , \quad \sin\alpha_1 = \frac{4}{5} \quad , \quad \alpha_2 = \arccos(2\frac{\sqrt{4}}{5}) \quad , \quad \sin\alpha_2 = \frac{3}{5}$$

$$\alpha_3 = \arccos(\frac{\sqrt{21}}{5}) \quad , \quad \sin\alpha_3 = \frac{2}{5} \quad , \quad \alpha_4 = \arccos(\frac{2\sqrt{6}}{5}) \quad , \quad \sin\alpha_4 = \frac{1}{5}$$

$$\begin{aligned} \Delta M(\vec{\phi}^{TK1}) &= (-\frac{3}{\pi} + \frac{1}{2\sqrt{3}})\hbar m - (\frac{15}{\pi} - \frac{1}{\pi}(\sqrt{9}\arccos(\frac{\sqrt{9}}{5}) + 2\sqrt{4}\arccos(\frac{2\sqrt{4}}{5}) + \\ &+ \sqrt{21}\arccos(\frac{\sqrt{21}}{5}) + 2\sqrt{6}\arccos(\frac{2\sqrt{6}}{5})))\hbar m = -2.82180\hbar m \quad . \end{aligned}$$

Thence, we obtain:

(2.8218*6.582119*10^-16*2.16e+9 + 2.24628*6.582119*10^-16*2.16e+9 + 1.75076*6.582119*10^-16*2.16e+9 + 1.33251*6.582119*10^-16*2.16e+9 + 0.984564* 6.582119*10^-16*2.16e+9)

Input interpretation

$$2.8218 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 2.24628 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 1.75076 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 1.33251 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 0.984564 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9$$

Result

0.0000129888733943014556

0.0000129888733943014556

Inverting:

1/(2.8218*6.582119*10^-16*2.16e+9 + 2.24628*6.582119*10^-16*2.16e+9 + 1.75076*6.582119*10^-16*2.16e+9 + 1.33251*6.582119*10^-16*2.16e+9 + 0.984564* 6.582119*10^-16*2.16e+9)

Input interpretation

$$1/(2.8218 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 2.24628 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 1.75076 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 1.33251 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9 + 0.984564 \times 6.582119 \times 10^{-16} \times 2.16 \times 10^9)$$

Result

76988.97122507368807219387593089266529633992833192647181828176140 5 …

76988.97122507….

From which:

1/((4e^(1/14)ln^(9/14)(3))/(ln^4(2)))(((1/(2.8218*6.582119e-16*2.16e+9 + 2.24628*6.582119e-16*2.16e+9 +1.75076*6.582119e-16*2.16e+9 + 1.33251*6.582119e-16*2.16e+9 + 0.984564*6.582119e-16*2.16e+9))))^2

Input interpretation

$$\frac{1}{\frac{4\sqrt[14]{e}\,\log^{9/14}(3)}{\log^4(2)}}\left(1/\left(2.8218\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 2.24628\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.75076\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.33251\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 0.984564\times 6.582119\times 10^{-16}\times 2.16\times 10^9\right)\right)^2$$

$\log(x)$ is the natural logarithm

Result

$2.99792\ldots\times 10^8$

Result

$2.997924580882424582001219666040016883611648988748292050239 17\ldots\times 10^8$

2.99792458…*10^8 = c

4372-sqrt(((1/(2.8218*6.582119e-16*2.16e+9 + 2.24628*6.582119e-16*2.16e+9 +1.75076*6.582119e-16*2.16e+9 + 1.33251*6.582119e-16*2.16e+9 + 0.984564*6.582119e-16*2.16e+9))))+φ

Input interpretation

$$4372 - \sqrt{\left(1/\left(2.8218\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 2.24628\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.75076\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.33251\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 0.984564\times 6.582119\times 10^{-16}\times 2.16\times 10^9\right)\right)} + \phi$$

ϕ is the golden ratio

Result

4096.149…

4096.149…. ≈ 4096 = 64^2

27sqrt(4372-sqrt(((1/(2.8218*6.582119e-16*2.16e+9 + 2.24628*6.582119e-16*2.16e+9 +1.75076*6.582119e-16*2.16e+9 + 1.33251*6.582119e-16*2.16e+9 + 0.984564*6.582119e-16*2.16e+9))))+φ)+1

Input interpretation

$$27 \sqrt{\left(4372 - \sqrt{\left(1/\left(2.8218\times 6.582119\times 10^{-16}\times 2.16\times 10^{9} + 2.24628\times 6.582119\times 10^{-16}\times 2.16\times 10^{9} + 1.75076\times 6.582119\times 10^{-16}\times 2.16\times 10^{9} + 1.33251\times 6.582119\times 10^{-16}\times 2.16\times 10^{9} + 0.984564\times 6.582119\times 10^{-16}\times 2.16\times 10^{9}\right)\right)} + \phi\right)} + 1$$

ϕ is the golden ratio

Result

1729.031…

1729.031….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(27sqrt(4372-sqrt(((1/(2.8218*6.582119e-16*2.16e+9 + 2.24628*6.582119e-16*2.16e+9 +1.75076*6.582119e-16*2.16e+9 + 1.33251*6.582119e-16*2.16e+9 + 0.984564*6.582119e-16*2.16e+9))))+φ)+1)^1/15

Input interpretation

$$\left(27\sqrt{\left(4372-\sqrt{(1/(2.8218\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 2.24628\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.75076\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 1.33251\times 6.582119\times 10^{-16}\times 2.16\times 10^9 + 0.984564\times 6.582119\times 10^{-16}\times 2.16\times 10^9))}+\phi\right)}+1\right)\text{^}(1/15)$$

ϕ is the golden ratio

Result

1.6438172...

1.6438172…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

Ramanujan's integral

Now, we analyze the following fundamental integral developed by Srinivasa Ramanujan:

$$\int_{-\infty}^{\infty} \frac{J_{\mu+\xi}(x)}{x^{\mu+\xi}} \frac{J_{\nu-\xi}(y)}{y^{\nu-\xi}} e^{i t \xi}\, d\xi$$
$$= \left[\frac{2\cos\left(\frac{1}{2}t\right)}{x^2 e^{-i t/2} + y^2 e^{i t/2}}\right]^{(\mu+\nu)/2}$$
$$\times J_{\mu+\nu}\left[\sqrt{2\cos\left(\frac{1}{2}t\right)\left(x^2 e^{-i t/2} + y^2 e^{i t/2}\right)}\right] e^{i t(\nu-\mu)/2},$$

where $J_n(z)$ is a Bessel function of the first kind.

More information »

For J = 3:

[(2cos(1/2 t))/(x^2 e^(-it/2) + y^2 e^(it/2))]^((3)/2) ×J_3 [sqrt(2cos(1/2 t)(x^2 e^(-it/2) + y^2 e^(it/2)))] e^(it(1)/2)

Input

$$\left(\frac{2\cos\left(\frac{1}{2}t\right)}{x^2 e^{-i\times t/2} + y^2 e^{(i\,t)/2}}\right)^{3/2} J_3\left(\sqrt{2\cos\left(\frac{1}{2}t\right)\left(x^2 e^{-i\times t/2} + y^2 e^{(i\,t)/2}\right)}\right) e^{i\,t\times 1/2}$$

$J_n(z)$ is the Bessel function of the first kind

i is the imaginary unit

Exact result

$$2\sqrt{2}\, e^{(i\,t)/2}\left(\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{\left(e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2\right)\cos\left(\frac{t}{2}\right)}\right)$$

Alternate forms

$$e^{(i\,t)/2}\left(\frac{1+e^{i\,t}}{x^2 + e^{i\,t}y^2}\right)^{3/2} J_3\left(\sqrt{\left(1+e^{-i\,t}\right)x^2 + \left(1+e^{i\,t}\right)y^2}\right)$$

$$e^{(i\,t)/2}\left(\frac{e^{-(i\,t)/2} + e^{(i\,t)/2}}{e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2}\right)^{3/2} J_3\left(\sqrt{\left(e^{-(i\,t)/2} + e^{(i\,t)/2}\right)\left(e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2\right)}\right)$$

$$\frac{2\sqrt{2}\cos\left(\frac{t}{2}\right)\sqrt{\frac{e^{(i\,t)/2}\cos\left(\frac{t}{2}\right)}{x^2+e^{i\,t}y^2}}\, J_3\left(\sqrt{2}\sqrt{\left(e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2\right)\cos\left(\frac{t}{2}\right)}\right)}{y^2} - \frac{2\sqrt{2}\,x^2\cos\left(\frac{t}{2}\right)\sqrt{\frac{e^{(i\,t)/2}\cos\left(\frac{t}{2}\right)}{x^2+e^{i\,t}y^2}}\, J_3\left(\sqrt{2}\sqrt{\left(e^{-(i\,t)/2}x^2 + e^{(i\,t)/2}y^2\right)\cos\left(\frac{t}{2}\right)}\right)}{y^2\left(x^2+e^{i\,t}y^2\right)}$$

Expanded form

$$\frac{2\sqrt{2}\ e^{(i\,t)/2}\cos\left(\frac{t}{2}\right)\sqrt{\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2}}\ J_3\left(\sqrt{2}\ \sqrt{\left(e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2\right)\cos\left(\frac{t}{2}\right)}\right)}{e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2}$$

Alternate forms assuming t, x, and y are positive

$$\left(2\sqrt{2}\ \cos^{3/2}\left(\frac{t}{2}\right)\exp\left(\frac{1}{2}\,i\left(t+2\,\pi\left\lfloor\frac{\arg\left(e^{-(i\,t)/2}\left(x^2+e^{i\,t}\,y^2\right)\right)-\arg\left(\cos\left(\frac{t}{2}\right)\right)+\pi}{2\,\pi}\right\rfloor\right)\right) J_3\left((-1)^{\left\lfloor-\left(\arg\left(e^{-(i\,t)/2}\left(x^2+e^{i\,t}\,y^2\right)\right)+\arg\left(\cos\left(\frac{t}{2}\right)\right)-\pi\right)/(2\,\pi)\right\rfloor}\sqrt{2}\ \sqrt{e^{-(i\,t)/2}\left(x^2+e^{i\,t}\,y^2\right)}\ \sqrt{\cos\left(\frac{t}{2}\right)}\right)\right)\Big/\left(e^{-(i\,t)/2}\left(x^2+e^{i\,t}\,y^2\right)\right)^{3/2}$$

$$\left(2\sqrt{2}\ \cos^{3/2}\left(\frac{t}{2}\right)\exp\left(i\,\pi\left\lfloor\frac{\arg\left(e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2\right)}{2\,\pi}-\frac{\arg\left(\cos\left(\frac{t}{2}\right)\right)}{2\,\pi}+\frac{1}{2}\right\rfloor+\frac{i\,t}{2}\right) J_3\left(\sqrt{2}\ \exp\left(i\,\pi\left\lfloor-\frac{\arg\left(e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2\right)}{2\,\pi}-\frac{\arg\left(\cos\left(\frac{t}{2}\right)\right)}{2\,\pi}+\frac{1}{2}\right\rfloor\right)\sqrt{e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2}\ \sqrt{\cos\left(\frac{t}{2}\right)}\right)\right)\Big/\left(e^{-(i\,t)/2}\,x^2+e^{(i\,t)/2}\,y^2\right)^{3/2}$$

Derivative

$$\frac{\partial}{\partial x}\left(\left(\frac{2\cos\left(\frac{t}{2}\right)}{x^2 e^{-(i t)/2} + y^2 e^{(i t)/2}}\right)^{3/2} J_3\left(\sqrt{2\cos\left(\frac{t}{2}\right)\left(x^2 e^{-(i t)/2} + y^2 e^{(i t)/2}\right)}\right) e^{(i t)/2}\right) =$$

$$\left(2 x \cos\left(\frac{t}{2}\right)\left(\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i t)/2} x^2 + e^{(i t)/2} y^2}\right)^{3/2} \left(J_2\left(\sqrt{2}\sqrt{\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)\cos\left(\frac{t}{2}\right)}\right) - J_4\left(\sqrt{2}\sqrt{\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)\cos\left(\frac{t}{2}\right)}\right)\right)\right) \Big/ \left(\sqrt{\cos\left(\frac{t}{2}\right)\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)}\right) - \frac{6\sqrt{2}\, x\cos\left(\frac{t}{2}\right)\sqrt{\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i t)/2} x^2 + e^{(i t)/2} y^2}}\, J_3\left(\sqrt{2}\sqrt{\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)\cos\left(\frac{t}{2}\right)}\right)}{\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)^2}$$

For t = 4π:

$$2\sqrt{2}\, e^{(i t)/2}\left(\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i t)/2} x^2 + e^{(i t)/2} y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{\left(e^{-(i t)/2} x^2 + e^{(i t)/2} y^2\right)\cos\left(\frac{t}{2}\right)}\right)$$

2 sqrt(2) e^((i*4Pi)/2) (cos((4Pi)/2)/(e^(-(i (4Pi))/2) x^2 + e^((i (4Pi))/2) y^2))^(3/2) J_3(sqrt(2) sqrt((e^(-(i (4Pi))/2) x^2 + e^((i (4Pi))/2) y^2) cos((4Pi)/2)))

Input

$$2\sqrt{2}\, e^{1/2 (i\times 4\pi)}\left(\frac{\cos\left(\frac{4\pi}{2}\right)}{e^{-1/2 (i (4\pi))} x^2 + e^{1/2 (i (4\pi))} y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{\left(e^{-1/2 (i (4\pi))} x^2 + e^{1/2 (i (4\pi))} y^2\right)\cos\left(\frac{4\pi}{2}\right)}\right)$$

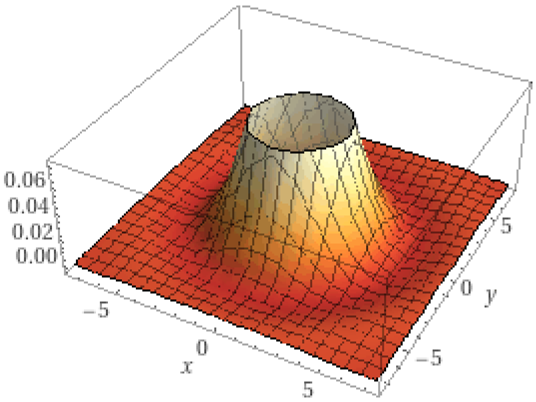
$J_n(z)$ is the Bessel function of the first kind

i is the imaginary unit

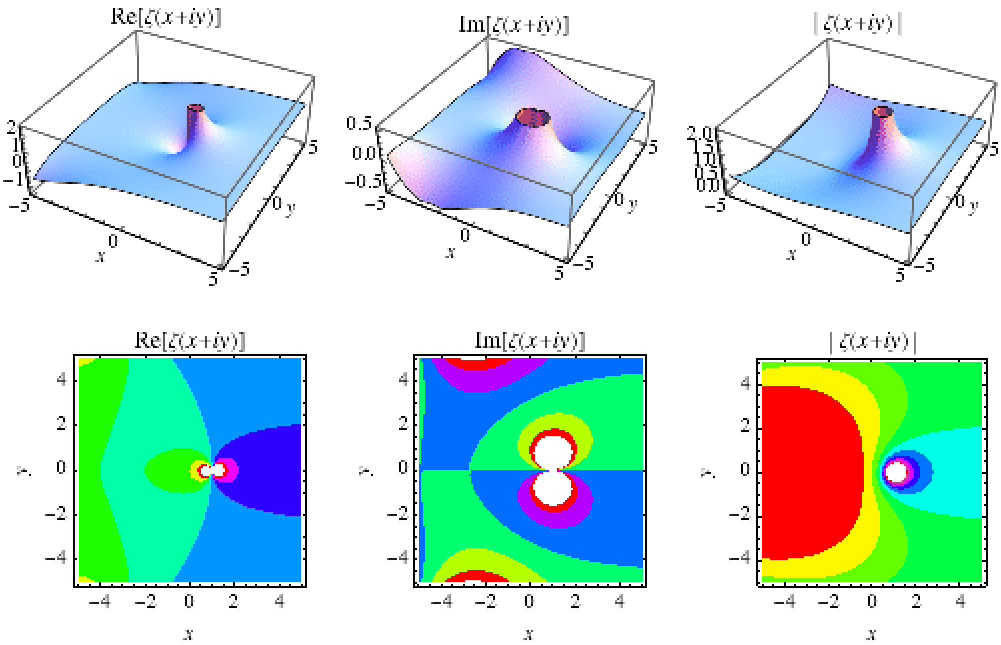
Exact result

$$2\sqrt{2}\left(\frac{1}{x^2+y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)$$

3D plot

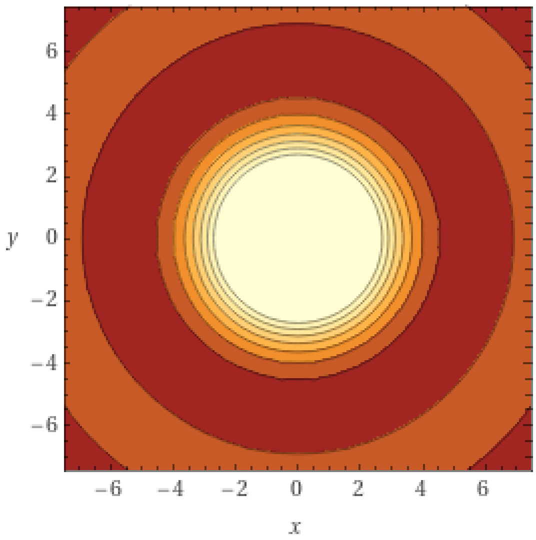


We note the following connection with the below plots concerning the Riemann zeta function, principally with the second figure:



https://mathworld.wolfram.com/RiemannZetaFunctionZeros.html

Contour plot



Alternate form assuming x and y are real

$${}_0\tilde{F}_1\left(;4;-\frac{x^2}{2}-\frac{y^2}{2}\right)$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternate form assuming x and y are positive

$$\frac{2\sqrt{2}\,J_3\left(\sqrt{2}\,\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^{3/2}}$$

Series expansion at x=0

$$2\sqrt{2}\left(\frac{1}{y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{y^2}\right) - \frac{2x^2\left(\sqrt{\frac{1}{y^2}}\sqrt{y^2}\,J_4\left(\sqrt{2}\sqrt{y^2}\right)\right)}{y^4} + \frac{x^4\left(4\sqrt{2}\sqrt{y^2}\,J_4\left(\sqrt{2}\sqrt{y^2}\right) - y^2 J_3\left(\sqrt{2}\sqrt{y^2}\right)\right)}{\sqrt{2}\sqrt{\frac{1}{y^2}}\,y^8} + O\left(x^6\right)$$

(Taylor series)

Partial derivatives

$$\frac{\partial}{\partial x}\left(2\sqrt{2}\left(\frac{1}{x^2+y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right) = 2x\sqrt{\frac{1}{x^2+y^2}}\left(\frac{J_2\left(\sqrt{2}\sqrt{x^2+y^2}\right) - J_4\left(\sqrt{2}\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^{3/2}} - \frac{3\sqrt{2}\,J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^2}\right)$$

$$\frac{\partial}{\partial y}\left(2\sqrt{2}\left(\frac{1}{x^2+y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right) = 2y\sqrt{\frac{1}{x^2+y^2}}\left(\frac{J_2\left(\sqrt{2}\sqrt{x^2+y^2}\right) - J_4\left(\sqrt{2}\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^{3/2}} - \frac{3\sqrt{2}\,J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^2}\right)$$

For x = 4 and y = 8 :

$$2\sqrt{2}\left(\frac{1}{x^2+y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)$$

2 sqrt(2) (1/(4^2 + 8^2))^(3/2) BesselJ(3, sqrt(2) sqrt(4^2 + 8^2))

Input

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{J_3(4\sqrt{10})}{80\sqrt{10}}$$

Decimal approximation

0.0003136757872854398166470437153726744598780069518421688980189654
…

0.00031367578….

Alternate form

${}_0\tilde{F}_1(;4;-40)$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = 2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\Gamma(4)}$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{2 I_3\left(i\sqrt{2}\sqrt{4^2+8^2}\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{4^2+8^2}\right)^3}$$

Series representations

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{1}{6}\sum_{k=0}^{\infty}\frac{(-40)^k}{k!\,(4)_k}$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}{80\sqrt{10}}$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{\sum_{k=0}^{\infty}\frac{\left(4\sqrt{10}-z_0\right)^k J_3^{(0,k)}(z_0)}{k!}}{80\sqrt{10}}$$

Integral representations

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{16}{15\pi}\int_0^1\left(1-t^2\right)^{5/2}\cos\left(4\sqrt{10}\,t\right)dt$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = \frac{1}{80\sqrt{10}\,\pi}\int_0^{\pi}\cos\left(3t-4\sqrt{10}\sin(t)\right)dt$$

$$2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right) = -\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-40/t+t}}{t^4}\,dt \quad \text{for } \gamma > 0$$

Inverting:

2 sqrt(2) (1/(4^2 + 8^2))^(3/2) BesselJ(3, sqrt(2) sqrt(4^2 + 8^2))

Input

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{80\sqrt{10}}{J_3(4\sqrt{10})}$$

Decimal approximation

3188.005069355309906446092560464253115809013103473045840985072573 7
…

3188.0050693553….

Alternate form

$$\frac{1}{{}_0\tilde{F}_1(;4;-40)}$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{1}{\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\Gamma(4)}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{1}{\frac{2 I_3\left(i\sqrt{2}\sqrt{4^2+8^2}\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{4^2+8^2}\right)^3}}$$

Series representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{6}{\sum_{k=0}^{\infty}\frac{(-40)^k}{k!\,(4)_k}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{80\sqrt{10}}{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{80\sqrt{10}}{\sum_{k=0}^{\infty} \frac{(4\sqrt{10}-z_0)^k J_3^{(0,k)}(z_0)}{k!}}$$

Integral representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{80\sqrt{10}\,\pi}{\int_0^{\pi}\cos\left(3t-4\sqrt{10}\sin(t)\right)dt}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{15\pi}{16\int_0^1 (1-t^2)^{5/2}\cos\left(4\sqrt{10}\,t\right)dt}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)} = \frac{2i\pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-40/t+t}}{t^4}\,dt} \quad \text{for } \gamma > 0$$

For x = 8 and y = 16

2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2))

Input

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{J_3(8\sqrt{10})}{640\sqrt{10}}$$

Decimal approximation

0.000033992351896332677172487053144953764209267234815256918480748 5

…

0.000033992351….

Alternate form

${}_0\tilde{F}_1(;4;-160)$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = 2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\Gamma(4)}$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{2\,I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}$$

Series representations

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{1}{6}\sum_{k=0}^{\infty}\frac{(-160)^k}{k!\,(4)_k}$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}{640\sqrt{10}}$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{\sum_{k=0}^{\infty}\frac{\left(8\sqrt{10}-z_0\right)^k J_3^{(0,k)}(z_0)}{k!}}{640\sqrt{10}}$$

Integral representations

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{16}{15\pi}\int_0^1\left(1-t^2\right)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-160/t+t}}{t^4}\,dt \quad \text{for } \gamma > 0$$

$$2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{1}{640\sqrt{10}\,\pi}\int_0^{\pi}\cos\left(3t-8\sqrt{10}\sin(t)\right)dt$$

Inverting:

1/(((2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2)))))

Input

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{640\sqrt{10}}{J_3\left(8\sqrt{10}\right)}$$

Decimal approximation

29418.382201082317713783427691816103831845858954446570792731463201
…

29418.38220108….

Alternate form

$$\frac{1}{{}_0\tilde{F}_1(;4;-160)}$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{1}{\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\Gamma(4)}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{1}{\frac{2 I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}}$$

Series representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{6}{\sum_{k=0}^{\infty}\frac{(-160)^k}{k!\,(4)_k}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{640\sqrt{10}}{\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{640\sqrt{10}}{\sum_{k=0}^{\infty}\frac{\left(8\sqrt{10}-z_0\right)^k J_3^{(0,k)}(z_0)}{k!}}$$

Integral representations

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{640\sqrt{10}\,\pi}{\int_0^\pi \cos\left(3t - 8\sqrt{10}\sin(t)\right)dt}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{15\pi}{16\int_0^1 (1-t^2)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt}$$

$$\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} = \frac{2i\pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-160/t+t}}{t^4}\,dt} \quad \text{for } \gamma > 0$$

Dividing the two results:

[(((1/(((2 sqrt(2) (1/(8^2+16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2+16^2))))))))1/(((1/((2 sqrt(2) (1/(4^2+8^2))^(3/2) BesselJ(3, sqrt(2) sqrt(4^2+8^2)))))))]^(1/(4+(((√(10-2√5) -2))⁄((√5-1)))^(1/3)))

Input

$$\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} \times \frac{1}{\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}}\right)^{\wedge}\left(\frac{1}{4+\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$4+\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{8\,J_3(4\sqrt{10})}{J_3(8\sqrt{10})}}$$

Decimal approximation

1.6114606070210747091179671891757946059187491743120132747527655332
...

1.611460607…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms

$$\text{root of } x^{12}+2x^9-6x^6-2x^3+1 \text{ near } x=0.657375 + 4\sqrt{\frac{{}_0\tilde{F}_1(;4;-40)}{{}_0\tilde{F}_1(;4;-160)}}$$

$$4+\sqrt[3]{\frac{\sqrt{2(5-\sqrt{5})}-2}{\sqrt{5}-1}}\sqrt{\frac{8\,J_3(4\sqrt{10})}{J_3(8\sqrt{10})}}$$

$$\left(\frac{8\,J_3(4\sqrt{10})}{J_3(8\sqrt{10})}\right)^{\text{root of } 16228481x^{12}-49115232x^{11}+68003352x^{10}-56975874x^9+32180832x^8-12911472x^7+3773946x^6-809856x^5+126648x^4-14078x^3+1056x^2-48x+1 \text{ near } x=0.214713}$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$4+3\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{1}{\frac{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}}} =$$

$$4+3\sqrt[3]{\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}}\sqrt{\frac{1}{\frac{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}}}$$

$$4+3\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{1}{\frac{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}}} =$$

$$4+3\sqrt[3]{\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}}\sqrt{\frac{1}{\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\frac{\left(2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}\right)\Gamma(4)}{\Gamma(4)}}}}$$

$$4+3\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{1}{\frac{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}}} =$$

$$4+3\sqrt[3]{\frac{-2+\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}}}\sqrt{\frac{1}{\frac{2\,I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\frac{\left(2\,I_3\left(i\sqrt{2}\sqrt{4^2+8^2}\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}\right)\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}{\left(i\sqrt{2}\sqrt{4^2+8^2}\right)^3}}}}$$

Series representations

$$4+\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} = 4+\sqrt[3]{\frac{-2+\sqrt{2\left(5-\sqrt{5}\right)}}{-1+\sqrt{5}}}\sqrt{\frac{\sum_{k=0}^{\infty}\frac{(-40)^k}{k!\,(4)_k}}{\sum_{k=0}^{\infty}\frac{(-160)^k}{k!\,(4)_k}}}$$

$$4+\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$4+\sqrt[3]{\frac{-2+\sqrt{2\left(5-\sqrt{5}\right)}}{-1+\sqrt{5}}}\sqrt{8}\;4+\sqrt[3]{\frac{-2+\sqrt{2\left(5-\sqrt{5}\right)}}{-1+\sqrt{5}}}\sqrt{\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}}$$

$$4+\sqrt[3]{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\sqrt{\frac{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2}J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$4+\sqrt[3]{\frac{-2+\sqrt{2\left(5-\sqrt{5}\right)}}{-1+\sqrt{5}}}\sqrt{8}\;4+\sqrt[3]{\frac{-2+\sqrt{2\left(5-\sqrt{5}\right)}}{-1+\sqrt{5}}}\sqrt{\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{40^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{160^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}}$$

We obtain also:

((([(((1/(((2 sqrt(2) (1/(8^2+16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2+16^2)))))))))+(((1/((2 sqrt(2) (1/(4^2+8^2))^(3/2) BesselJ(3, sqrt(2) sqrt(4^2+8^2))))))))]))^1/21

Input

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} + \frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}}$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}}$$

Decimal approximation

1.640284478620800727276954920065347078241266858263683894793909 6177

...

1.6402844786…. ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... (trace of the instanton shape)

Alternate forms

$$\sqrt[21]{\frac{1}{{}_0\tilde{F}_1(;4;-160)} + \frac{1}{{}_0\tilde{F}_1(;4;-40)}}$$

$$2^{3/14}\sqrt[14]{5}\sqrt[21]{\frac{1}{J_3(4\sqrt{10})} + \frac{8}{J_3(8\sqrt{10})}}$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

All 21st roots of (80 sqrt(10))/(J_3(4 sqrt(10))) + (640 sqrt(10))/(J_3(8 sqrt(10)))

$$e^0 \sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}} \approx 1.6403$$ (real, principal root)

$$e^{(2 i \pi)/21} \sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}} \approx 1.5674 + 0.4835\, i$$

$$e^{(4 i \pi)/21} \sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}} \approx 1.3553 + 0.9240\, i$$

$$e^{(2 i \pi)/7} \sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}} \approx 1.0227 + 1.2824\, i$$

$$e^{(8 i \pi)/21} \sqrt[21]{\frac{80\sqrt{10}}{J_3(4\sqrt{10})} + \frac{640\sqrt{10}}{J_3(8\sqrt{10})}} \approx 0.5993 + 1.5269\, i$$

Alternative representations

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} + \frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$\left(\frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}} + \frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}\right)^\wedge (1/21)$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} + \frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$\left(\frac{1}{\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^2\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\Gamma(4)}} + \frac{1}{\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\Gamma(4)}}\right)^\wedge (1/21)$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$\left(\frac{1}{\frac{2 I_3\left(i\sqrt{2}\sqrt{4^2+8^2}\right)\left(\frac{1}{4^2+8^2}\right)^{3/2}\left(\sqrt{2}\sqrt{4^2+8^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{4^2+8^2}\right)^3}}+\frac{1}{\frac{2 I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}}\right)\hat{}\,(1/21)$$

Series representations

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$2^{3/14}\sqrt[14]{5}\sqrt[21]{\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k 2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}+\frac{8}{\sum_{k=0}^{\infty}\frac{(-1)^k 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}}$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$2^{3/14}\sqrt[14]{5}\sqrt[21]{\frac{8\sum_{k=0}^{\infty}\frac{(-1)^k 2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}+\sum_{k=0}^{\infty}\frac{(-1)^k 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{9/2+3k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}}$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$2^{3/14}\sqrt[14]{5}\sqrt[21]{\frac{1}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{40^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}+\frac{8}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{160^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}}$$

Integral representations

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$2^{3/14}\sqrt[14]{5}\sqrt[21]{\pi}\sqrt[21]{\frac{\int_0^{\pi}\cos\left(3t-8\sqrt{10}\sin(t)\right)dt+8\int_0^{\pi}\cos\left(3t-4\sqrt{10}\sin(t)\right)dt}{\left(\int_0^{\pi}\cos\left(3t-8\sqrt{10}\sin(t)\right)dt\right)\int_0^{\pi}\cos\left(3t-4\sqrt{10}\sin(t)\right)dt}}$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$\frac{\sqrt[21]{15\pi}\sqrt[21]{\frac{\int_0^{\pi}\cos\left(4\sqrt{10}\cos(t)\right)\sin^6(t)\,dt+\int_0^{\pi}\cos\left(8\sqrt{10}\cos(t)\right)\sin^6(t)\,dt}{\left(\int_0^{\pi}\cos\left(4\sqrt{10}\cos(t)\right)\sin^6(t)\,dt\right)\int_0^{\pi}\cos\left(8\sqrt{10}\cos(t)\right)\sin^6(t)\,dt}}}{\sqrt[7]{2}}$$

$$\sqrt[21]{\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}+\frac{1}{2\sqrt{2}\left(\frac{1}{4^2+8^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{4^2+8^2}\right)}} =$$

$$\frac{\sqrt[21]{15\pi}\sqrt[21]{\frac{\int_0^{1}(1-t^2)^{5/2}\cos\left(4\sqrt{10}\,t\right)dt+\int_0^{1}(1-t^2)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt}{\left(\int_0^{1}(1-t^2)^{5/2}\cos\left(4\sqrt{10}\,t\right)dt\right)\int_0^{1}(1-t^2)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt}}}{2^{4/21}}$$

1/((sqrt(1/3 (-27 + 9 e + 7 π + 8 log(2))))) (((1/(((2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2)))))))^2

Input

$$\frac{1}{\sqrt{\frac{1}{3}\left(-27+9 e+7 \pi+8 \log (2)\right)}}\left(\frac{1}{2 \sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2} \sqrt{8^2+16^2}\right)}\right)^2$$

$\log(x)$ is the natural logarithm

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{4096000 \sqrt{\frac{3}{-27+9 e+7 \pi+8 \log (2)}}}{J_3\left(8 \sqrt{10}\right)^2}$$

Decimal approximation

$2.997924585672935410302003062750254841244235440748485927870 95\ldots \times 10^8$

$2.99792458\ldots*10^8 = c$

Alternate forms

$$\frac{\sqrt{\frac{3}{-27+9 e+7 \pi+\log (256)}}}{{}_0\tilde{F}_1(;4;-160)^2}$$

$$\frac{4096000 \sqrt{\frac{3}{-27+9 e+7 \pi+\log (256)}}}{J_3\left(8 \sqrt{10}\right)^2}$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log(2)\right)}} = \frac{\left(\frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log(2)\right)}}$$

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log(2)\right)}} = \frac{\left(\frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log_e(2)\right)}}$$

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log(2)\right)}} = \frac{\left(\frac{1}{2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}\right)^2}{\sqrt{\frac{1}{3}\left(-27+9e+7\pi+8\log(a)\log_a(2)\right)}}$$

Series representations

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}(-27+9e+7\pi+8\log(2))}} = 8192000 \Big/ \left(\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(-9+3e+\frac{7\pi}{3}-x+\frac{8\log(2)}{3}\right)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-9+3e+\frac{7\pi}{3}-x+\frac{8\log(2)}{3}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-3-2k}\left(\sqrt{2}\sqrt{320}\right)^{3+2k}}{k!\,\Gamma(4+k)}\right)^2\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}(-27+9e+7\pi+8\log(2))}} = 8192000 \Big/ \left(\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{1}{3}(-27+9e+7\pi+8\log(2))\right)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{1}{3}(-27+9e+7\pi+8\log(2))\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-3-2k}\left(\sqrt{2}\sqrt{320}\right)^{3+2k}}{k!\,\Gamma(4+k)}\right)^2\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)}\right)^2}{\sqrt{\frac{1}{3}(-27+9e+7\pi+8\log(2))}} = \left(524\,288\,000\,\Gamma(4)^2\right) \Big/$$

$$\left(\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(-9+3e+\frac{7\pi}{3}-x+\frac{8\log(2)}{3}\right)}{2\pi}\right\rfloor\right)\right.$$

$$\sqrt{2}^6\sqrt{320}^6\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-9+3e+\frac{7\pi}{3}-x+\frac{8\log(2)}{3}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(\sqrt{2}\sqrt{320}\right)^{2k}}{k!\,(4)_k}\right)^2\right) \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

1/17 1/(((2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2))))) - 5((((√(10-2√5) -2))⁄((√5-1))))

Input

$$\frac{1}{17}\times\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} - 5\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\frac{640\sqrt{10}}{17 J_3\left(8\sqrt{10}\right)} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}$$

Decimal approximation

1729.072675432698980507118993297805182792536262760510770482172610 2…

1729.072675432….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{5}{2}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{640\sqrt{10}}{17\,J_3\left(8\sqrt{10}\right)}$$

$$-\frac{5\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}{\sqrt{5}-1}-\frac{640\left(\sqrt{10}-5\sqrt{2}\right)}{17\left(\sqrt{5}-1\right)J_3\left(8\sqrt{10}\right)}$$

$$\frac{5\left(640\sqrt{2}-128\sqrt{10}+34\,J_3\left(8\sqrt{10}\right)-17\sqrt{2\left(5-\sqrt{5}\right)}\,J_3\left(8\sqrt{10}\right)\right)}{17\left(\sqrt{5}-1\right)J_3\left(8\sqrt{10}\right)}$$

Expanded forms

$$\frac{5}{2}+\frac{5\sqrt{5}}{2}-\frac{5}{4}\sqrt{10-2\sqrt{5}}-\frac{5}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{640\sqrt{10}}{17\,J_3\left(8\sqrt{10}\right)}$$

$$\frac{10}{\sqrt{5}-1}-\frac{5\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}+\frac{640\sqrt{10}}{17\,J_3\left(8\sqrt{10}\right)}$$

Alternative representations

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{1}{17\left(2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}\right)} - \frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{1}{\frac{17\left(2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}\right)}{\Gamma(4)}} - \frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{1}{\frac{17\left(2\,I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}\right)}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}} - \frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}$$

Series representations

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{6}{17\sum_{k=0}^{\infty}\frac{(-160)^k}{k!\,(4)_k}}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{640\sqrt{10}}{17\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{640\sqrt{10}}{17\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{160^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}$$

Integral representations

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{640\sqrt{10}\,\pi}{17\int_0^{\pi}\cos\left(3t-8\sqrt{10}\sin(t)\right)dt}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{15\pi}{272\int_0^{1}\left(1-t^2\right)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt}$$

$$\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1} = \frac{10}{-1+\sqrt{5}} - \frac{5\sqrt{10-2\sqrt{5}}}{-1+\sqrt{5}} + \frac{15\pi}{136\int_0^{\pi}\cos\left(8\sqrt{10}\cos(t)\right)\sin^6(t)\,dt}$$

((1/17 1/(((2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2))))) - 5((((√(10-2√5) -2))⁄((√5-1))))))^1/15

Input

$$\sqrt[15]{\frac{1}{17}\times\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} - 5\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$\sqrt[15]{\frac{640\sqrt{10}}{17 J_3(8\sqrt{10})} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}}$$

Decimal approximation

1.643819834981221842573392370936626294220643847295624128971742737 9

...

1.64381983…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

Alternate form

$$\sqrt[15]{\frac{5}{2}\left(1+\sqrt{5}-\sqrt{2(5+\sqrt{5})}\right) + \frac{640\sqrt{10}}{17 J_3(8\sqrt{10})}}$$

All 15th roots of (640 sqrt(10))/(17 J_3(8 sqrt(10))) - (5 (sqrt(10 - 2 sqrt(5)) - 2))/(sqrt(5) - 1)

$$e^0 \sqrt[15]{\frac{640\sqrt{10}}{17 J_3(8\sqrt{10})} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} \approx 1.644 \text{ (real, principal root)}$$

$$e^{(2 i \pi)/15} \sqrt[15]{\frac{640\sqrt{10}}{17 J_3(8\sqrt{10})} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} \approx 1.502 + 0.6686 i$$

$$e^{(4 i \pi)/15} \sqrt[15]{\frac{640 \sqrt{10}}{17 J_3(8 \sqrt{10})} - \frac{5\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{\sqrt{5} - 1}} \approx 1.0999 + 1.2216 i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\frac{640 \sqrt{10}}{17 J_3(8 \sqrt{10})} - \frac{5\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{\sqrt{5} - 1}} \approx 0.5080 + 1.563 i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\frac{640 \sqrt{10}}{17 J_3(8 \sqrt{10})} - \frac{5\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{\sqrt{5} - 1}} \approx -0.1718 + 1.635 i$$

Alternative representations

$$\sqrt[15]{\frac{1}{\left(2 \sqrt{2} \left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2} \sqrt{8^2 + 16^2}\right)\right) 17} - \frac{5\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{\sqrt{5} - 1}} =$$

$$\left(\frac{1}{17\left(2\, {}_0\tilde{F}_1\left(; 4; -\frac{1}{4}\left(\sqrt{2} \sqrt{8^2 + 16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2} \sqrt{2} \sqrt{8^2 + 16^2}\right)^3 \sqrt{2}\right)} - \frac{5\left(-2 + \sqrt{10 - 2\sqrt{5}}\right)}{-1 + \sqrt{5}}\right) \text{^}\,(1/15)$$

$$\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} =$$

$$\left(\frac{1}{\frac{17\left(2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}\right)}{\Gamma(4)}} - \frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}\right)^{\wedge}(1/15)$$

$$\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} =$$

$$\sqrt[15]{\frac{1}{\frac{17\left(2 I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(\frac{1}{8^2+16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}\right)}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}} - \frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}}}$$

Series representations

$$\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} =$$

$$\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{640\sqrt{10}}{17\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}}$$

$$\frac{\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}}}{\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{640\sqrt{10}}{17\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{6+4k+1/2(3+2k)} \times 5^{1/2(3+2k)}}{k!\,\Gamma(4+k)}}}} =$$

$$\frac{\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}}}{\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{640\sqrt{10}}{17\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{3}{2}-j}\frac{160^{-s}\,\Gamma\left(\frac{3}{2}+s\right)}{\Gamma\left(\frac{5}{2}-s\right)}}}} =$$

Integral representations

$$\frac{\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}}}{\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{640\sqrt{10}\,\pi}{17\int_0^{\pi}\cos\left(3t-8\sqrt{10}\sin(t)\right)dt}}} =$$

$$\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} =$$
$$\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{15\pi}{272\int_0^1 (1-t^2)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt}}$$

$$\sqrt[15]{\frac{1}{\left(2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)\right)17} - \frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}} =$$
$$\sqrt[15]{-\frac{5\left(-2+\sqrt{10-2\sqrt{5}}\right)}{-1+\sqrt{5}} + \frac{15\pi}{136\int_0^{\pi}\cos\left(8\sqrt{10}\cos(t)\right)\sin^6(t)\,dt}}$$

(1/27((1/17 1/(((2 sqrt(2) (1/(8^2 + 16^2))^(3/2) BesselJ(3, sqrt(2) sqrt(8^2 + 16^2))))) - 5((((√(10-2√5) -2))⁄((√5-1)))))-1))^2+276 -1/2*Φ

Input

$$\left(\frac{1}{27}\left(\left(\frac{1}{17}\times\frac{1}{2\sqrt{2}\left(\frac{1}{8^2+16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)} - 5\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)-1\right)\right)^2 + 276 - \frac{1}{2}\Phi$$

$J_n(z)$ is the Bessel function of the first kind

Φ is the golden ratio conjugate

Exact result

$$-\frac{\Phi}{2}+276+\frac{1}{729}\left(-1-\frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}+\frac{640\sqrt{10}}{17\,J_3(8\sqrt{10})}\right)^2$$

Exact form

$$\frac{1-\phi}{2}+276+\frac{1}{729}\left(1+\frac{5\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}-\frac{640\sqrt{10}}{17\,J_3(8\sqrt{10})}\right)^2$$

ϕ is the golden ratio

Decimal approximation

4372.0355256354277886755525190859594172218917936619281555379307340
...

4372.03552563…. ≈ 4372

where 4372 is a value indicated in the fundamental Ramanujan paper "**Modular equations and Approximations to π**"

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}}-24+276e^{-\pi\sqrt{22}}-\cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}}+\cdots, \end{aligned}$$

so that

$$64(g_{22}^{24}+g_{22}^{-24})=e^{\pi\sqrt{22}}-24+4372e^{-\pi\sqrt{22}}+\cdots=64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}}=2508951.9982\ldots.$$

Alternate forms

$$-\frac{\Phi}{2}+276+\frac{1}{729}\left(\frac{3}{2}+\frac{5\sqrt{5}}{2}-5\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}+\frac{640\sqrt{10}}{17\,J_3\left(8\sqrt{10}\right)}\right)^2$$

$$-\frac{\Phi}{2}+\frac{201205}{729}+\frac{10\left(\sqrt{10-2\sqrt{5}}-2\right)}{729\left(\sqrt{5}-1\right)}+\frac{25\left(\sqrt{10-2\sqrt{5}}-2\right)^2}{729\left(\sqrt{5}-1\right)^2}+\frac{4096000}{210681\,J_3\left(8\sqrt{10}\right)^2}+\frac{-\frac{1280\sqrt{10}}{12393}-\frac{6400\sqrt{10}\left(\sqrt{10-2\sqrt{5}}-2\right)}{12393\left(\sqrt{5}-1\right)}}{J_3\left(8\sqrt{10}\right)}$$

$$\Big(-632043\,J_3\left(8\sqrt{10}\right)^2\Phi+210681\sqrt{5}\,J_3\left(8\sqrt{10}\right)^2\Phi+24576000-8192000\sqrt{5}+1305600\sqrt{2}\,J_3\left(8\sqrt{10}\right)-348160\sqrt{10}\,J_3\left(8\sqrt{10}\right)-1088000\sqrt{5-\sqrt{5}}\,J_3\left(8\sqrt{10}\right)+217600\sqrt{5\left(5-\sqrt{5}\right)}\,J_3\left(8\sqrt{10}\right)+348996400\,J_3\left(8\sqrt{10}\right)^2-116316720\sqrt{5}\,J_3\left(8\sqrt{10}\right)^2-31790\sqrt{2\left(5-\sqrt{5}\right)}\,J_3\left(8\sqrt{10}\right)^2+2890\sqrt{10\left(5-\sqrt{5}\right)}\,J_3\left(8\sqrt{10}\right)^2\Big)\Big/\left(210681\left(\sqrt{5}-1\right)^2 J_3\left(8\sqrt{10}\right)^2\right)$$

Expanded form

$$-\frac{\Phi}{2}+\frac{201205}{729}+\frac{350}{729\left(\sqrt{5}-1\right)^2}-\frac{50\sqrt{5}}{729\left(\sqrt{5}-1\right)^2}-\frac{100\sqrt{10-2\sqrt{5}}}{729\left(\sqrt{5}-1\right)^2}-\frac{20}{729\left(\sqrt{5}-1\right)}+\frac{10\sqrt{10-2\sqrt{5}}}{729\left(\sqrt{5}-1\right)}+\frac{4096000}{210681\,J_3\left(8\sqrt{10}\right)^2}-\frac{1280\sqrt{10}}{12393\,J_3\left(8\sqrt{10}\right)}+\frac{12800\sqrt{10}}{12393\left(\sqrt{5}-1\right)J_3\left(8\sqrt{10}\right)}-\frac{6400\sqrt{10\left(10-2\sqrt{5}\right)}}{12393\left(\sqrt{5}-1\right)J_3\left(8\sqrt{10}\right)}$$

For t = 2π :

$$2\sqrt{2}\, e^{(i\,t)/2} \left(\frac{\cos\left(\frac{t}{2}\right)}{e^{-(i\,t)/2}\, x^2 + e^{(i\,t)/2}\, y^2}\right)^{3/2} J_3\left(\sqrt{2}\, \sqrt{\left(e^{-(i\,t)/2}\, x^2 + e^{(i\,t)/2}\, y^2\right) \cos\left(\frac{t}{2}\right)}\right)$$

2 sqrt(2) e^((i*2Pi)/2) (cos((2Pi)/2)/(e^(-(i (2Pi))/2) x^2 + e^((i (2Pi))/2) y^2))^(3/2) J_3(sqrt(2) sqrt((e^(-(i (2Pi))/2) x^2 + e^((i (2Pi))/2) y^2) cos((2Pi)/2)))

Input

$$2\sqrt{2}\, e^{1/2\,(i\times 2\pi)} \left(\frac{\cos\left(\frac{2\pi}{2}\right)}{e^{-1/2\,(i\,(2\pi))}\, x^2 + e^{1/2\,(i\,(2\pi))}\, y^2}\right)^{3/2} J_3\left(\sqrt{2}\, \sqrt{\left(e^{-1/2\,(i\,(2\pi))}\, x^2 + e^{1/2\,(i\,(2\pi))}\, y^2\right) \cos\left(\frac{2\pi}{2}\right)}\right)$$

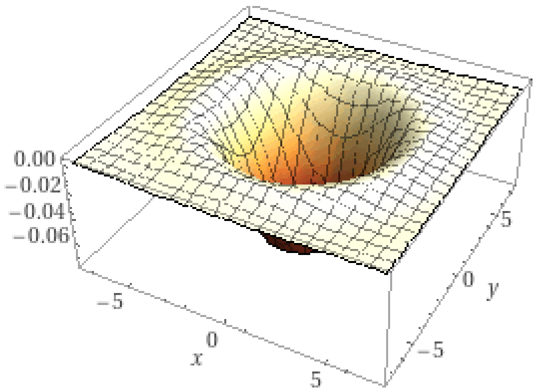
$J_n(z)$ is the Bessel function of the first kind

i is the imaginary unit

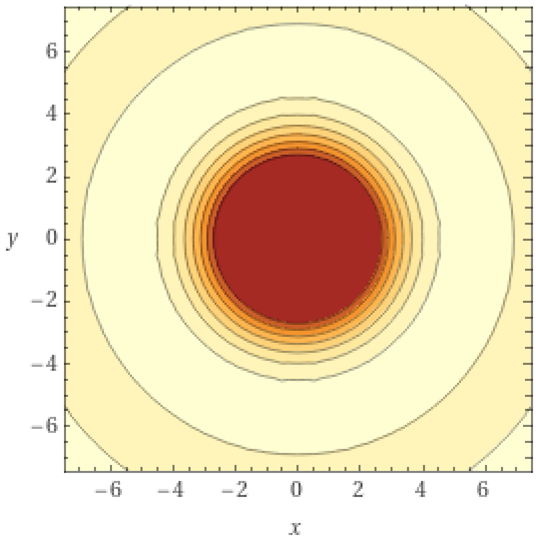
Exact result

$$-2\sqrt{2} \left(-\frac{1}{-x^2 - y^2}\right)^{3/2} J_3\left(\sqrt{2}\, \sqrt{x^2 + y^2}\right)$$

3D plot **(figures that can be related to the D-branes/Instantons)**



Contour plot



Alternate form assuming x and y are real

$$-{}_0\tilde{F}_1\left(;4;\frac{1}{2}\left(-x^2-y^2\right)\right)$$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternate form assuming x and y are positive

$$-\frac{2\sqrt{2}\,J_3\left(\sqrt{2}\,\sqrt{x^2+y^2}\right)}{\left(x^2+y^2\right)^{3/2}}$$

Expanded form

$$\frac{2\sqrt{2}\,\sqrt{-\frac{1}{-x^2-y^2}}\,J_3\left(\sqrt{2}\,\sqrt{x^2+y^2}\right)}{-x^2-y^2}$$

Series expansion at x=0

$$-2\left(\sqrt{2}\left(\frac{1}{y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{y^2}\right)\right)+\frac{2x^2\sqrt{\frac{1}{y^2}}\sqrt{y^2}\,J_4\left(\sqrt{2}\sqrt{y^2}\right)}{y^4}+\frac{x^4\left(y^2 J_3\left(\sqrt{2}\sqrt{y^2}\right)-4\sqrt{2}\sqrt{y^2}\,J_4\left(\sqrt{2}\sqrt{y^2}\right)\right)}{\sqrt{2}\sqrt{\frac{1}{y^2}}\,y^8}+O\left(x^6\right)$$

(Taylor series)

Partial derivatives

$$\frac{\partial}{\partial x}\left(-2\sqrt{2}\left(-\frac{1}{-x^2-y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right)=$$
$$\frac{1}{\left(x^2+y^2\right)^{5/2}}\,2x\sqrt{\frac{1}{x^2+y^2}}\left(-\left(x^2+y^2\right)J_2\left(\sqrt{2}\sqrt{x^2+y^2}\right)+3\sqrt{2}\sqrt{x^2+y^2}\,J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)+\left(x^2+y^2\right)J_4\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right)$$

$$\frac{\partial}{\partial y}\left(-2\sqrt{2}\left(-\frac{1}{-x^2-y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right)=$$
$$\frac{1}{\left(x^2+y^2\right)^{5/2}}\,2y\sqrt{\frac{1}{x^2+y^2}}\left(-\left(x^2+y^2\right)J_2\left(\sqrt{2}\sqrt{x^2+y^2}\right)+3\sqrt{2}\sqrt{x^2+y^2}\,J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)+\left(x^2+y^2\right)J_4\left(\sqrt{2}\sqrt{x^2+y^2}\right)\right)$$

For x = 2 and y = 4:

$$-2\sqrt{2}\left(-\frac{1}{-x^2-y^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{x^2+y^2}\right)$$

-2 sqrt(2) (-1/(-2^2 - 4^2))^(3/2) J_3(sqrt(2) sqrt(2^2 + 4^2))

Input

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$-\frac{J_3\left(2\sqrt{10}\right)}{10\sqrt{10}}$$

Decimal approximation

$-0.000526582141254937175640466925339938792096776393994932500321810\ldots$

-0.00052658214….

Alternate form

$-{}_0\tilde{F}_1(;4;-10)$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4+4^2}\right)^2\right)\left(-\frac{1}{-4-4^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4+4^2}\right)^3\sqrt{2}$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{2 I_3\left(i\sqrt{2}\sqrt{4+4^2}\right)\left(-\frac{1}{-4-4^2}\right)^{3/2}\left(\sqrt{2}\sqrt{4+4^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{4+4^2}\right)^3}$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{4+4^2}\right)^2\right)\left(-\frac{1}{-4-4^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{4+4^2}\right)^3\sqrt{2}}{\Gamma(4)}$$

Series representations

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{1}{6}\sum_{k=0}^{\infty}\frac{(-10)^k}{k!\,(4)_k}$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,10^{3/2+k}}{k!\,\Gamma(4+k)}}{10\sqrt{10}}$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{\sum_{k=0}^{\infty}\frac{\left(2\sqrt{10}-z_0\right)^k J_3^{(0,k)}(z_0)}{k!}}{10\sqrt{10}}$$

Integral representations

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = \frac{i}{2\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-10/t+t}}{t^4}\,dt \quad \text{for } \gamma > 0$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{16}{15\pi}\int_0^1\left(1-t^2\right)^{5/2}\cos\left(2\sqrt{10}\,t\right)dt$$

$$-2\sqrt{2}\left(-\frac{1}{-2^2-4^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{2^2+4^2}\right) = -\frac{1}{10\sqrt{10}\,\pi}\int_0^{\pi}\cos\left(3t-2\sqrt{10}\sin(t)\right)dt$$

For x = 8 and y = 16 :

-2 sqrt(2) (-1/(-8^2 - 16^2))^(3/2) J_3(sqrt(2) sqrt(8^2 + 16^2))

Input

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right)$$

$J_n(z)$ is the Bessel function of the first kind

Exact result

$$-\frac{J_3\left(8\sqrt{10}\right)}{640\sqrt{10}}$$

Decimal approximation

$-0.000033992351896332677172487053144953764209267234815256918480748$ ...

-0.0000339923….

Alternate form

$-{}_0\tilde{F}_1(;4;-160)$

${}_0\tilde{F}_1(;a;x)$ is the regularized hypergeometric function

Alternative representations

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -2\,{}_0\tilde{F}_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(-\frac{1}{-8^2-16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{2\,{}_0F_1\left(;4;-\frac{1}{4}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^2\right)\left(-\frac{1}{-8^2-16^2}\right)^{3/2}\left(\frac{1}{2}\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\Gamma(4)}$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{2\,I_3\left(i\sqrt{2}\sqrt{8^2+16^2}\right)\left(-\frac{1}{-8^2-16^2}\right)^{3/2}\left(\sqrt{2}\sqrt{8^2+16^2}\right)^3\sqrt{2}}{\left(i\sqrt{2}\sqrt{8^2+16^2}\right)^3}$$

Series representations

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{1}{6}\sum_{k=0}^{\infty}\frac{(-160)^k}{k!\,(4)_k}$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{15/2+5k}\times 5^{3/2+k}}{k!\,\Gamma(4+k)}}{640\sqrt{10}}$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{\sum_{k=0}^{\infty}\frac{\left(8\sqrt{10}-z_0\right)^k J_3^{(0,k)}(z_0)}{k!}}{640\sqrt{10}}$$

Integral representations

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = \frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-160/t+t}}{t^4}\,dt \quad \text{for } \gamma > 0$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{16}{15\pi}\int_0^1 \left(1-t^2\right)^{5/2}\cos\left(8\sqrt{10}\,t\right)dt$$

$$-2\sqrt{2}\left(-\frac{1}{-8^2-16^2}\right)^{3/2} J_3\left(\sqrt{2}\sqrt{8^2+16^2}\right) = -\frac{1}{640\sqrt{10}\,\pi}\int_0^\pi \cos\left(3t-8\sqrt{10}\sin(t)\right)dt$$

Mathematical connections with some sectors of String Theory

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{-\,2\,C} \;=\; \frac{h^2\left(p\;+\;1\;-\;\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\,e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\;-\;p)}$$

$$(A')^2 \;=\; k\,e^{-\,2\,A}\;+\;\frac{h^2}{16(p\;+\;1)}\,\left(7\;-\;p\;+\;\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\,e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 642, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18))   we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

from which:

$\frac{1}{4096}e^{-6C+\phi}$ = 1.6272016… * 10^-6

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757\ldots$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty}\frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.00666501784619000) = 2 i \pi \left\lfloor \frac{\arg(0.00666501784619000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.00666501784619000) = \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.00666501784619000) = \int_1^{0.00666501784619000} \frac{1}{t} \, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285, value very near to the ψ Regge slope 0.979:

Ψ	3	$m_c = 1500$	0.979	-0.09

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

From

AdS Vacua from Dilaton Tadpoles and Form Fluxes - *J. Mourad and A. Sagnotti* - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$e^{2C} = \frac{2\xi\, e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{\xi^7\, e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}\right)^7} \left[\frac{42}{\xi}\left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}\right) + 5\, T\, e^{2\phi}\right] . \qquad (2.7)$$

For

$T = \frac{16}{\pi^2}$

$\xi = 1$

we obtain:

(2*e^(0.989117352243/2)) / (1+sqrt(((1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))

Input interpretation:

$$\frac{2\, e^{0.989117352243/2}}{1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\, e^{2\times 0.989117352243}}}$$

Result:

0.83941881822... −
1.4311851867... i

Polar coordinates:

$r = 1.65919106525$ (radius), $\theta = -59.607521917°$ (angle)

1.65919106525….. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Series representations:

$$\frac{2\,e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}} = \frac{2\,e^{0.49455867612150000}}{1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{2\,e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}} = \frac{2\,e^{0.49455867612150000}}{1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{2\,e^{0.98911735224300000/2}}{1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}} = \frac{2\,e^{0.49455867612150000}}{1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right)^7} \left[\frac{42}{\xi}\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right) + 5 T e^{2\phi}\right]$$

we obtain:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]

Input interpretation:

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)$$

Result:

50.84107889... −
20.34506335... i

Polar coordinates:

$r = 54.76072411$ (radius), $\theta = -21.80979492°$ (angle)

54.76072411…..

Series representations:

$$\left(\left(42\left(1+\sqrt{1-\frac{16\, e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\, e^{2\times 0.9891173522430000}}{\pi^2}\right) e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\, e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 = \left(2\left(40\, e^{5.934704113458000}+21\, e^{3.956469408972000}\,\pi^2+21\, e^{3.956469408972000}\,\pi^2 \sqrt{-\frac{16\, e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(\pi^2\left(1+\sqrt{-\frac{16\, e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\, e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\, e^{2\times 0.9891173522430000}}{\pi^2}\right) e^{4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1-\frac{16\, e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7 = \left(2\left(40\, e^{5.934704113458000}+21\, e^{3.956469408972000}\,\pi^2+21\, e^{3.956469408972000}\,\pi^2 \sqrt{-\frac{16\, e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(\pi^2\left(1+\sqrt{-\frac{16\, e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.98911735224300000}}{\pi^2}\right)e^{4\times 0.98911735224300000}\right)\Big/\left(1+\sqrt{1-\frac{16\,e^{2\times 0.98911735224300000}}{3\,\pi^2}}\right)^7=$$

$$\left(2\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]*1/34

Input interpretation:

$$\frac{e^{4\times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}}\right)+5\times\frac{16}{\pi^2}\,e^{2\times 0.989117352243}\right)\times\frac{1}{34}$$

Result:

1.495325850… −
0.5983842161… i

Polar coordinates:

$r = 1.610609533$ (radius), $\theta = -21.80979492°$ (angle)

1.610609533…. result that is a good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right)e^{4\times 0.9891173522430000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(17\,\pi^2\left(1+\sqrt{-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{16}\right)^k\left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$\left(\left(42\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)+\frac{5\times 16\,e^{2\times 0.9891173522430000}}{\pi^2}\right) e^{4\times 0.9891173522430000}\right)\Big/\left(34\left(1+\sqrt{1-\frac{16\,e^{2\times 0.9891173522430000}}{3\,\pi^2}}\right)^7\right)=$$

$$\left(40\,e^{5.934704113458000}+21\,e^{3.956469408972000}\,\pi^2+21\,e^{3.956469408972000}\,\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(17\,\pi^2\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{16\,e^{1.978234704486000}}{3\,\pi^2}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Now, we have:

$$e^{2C} = \frac{2\,\xi\,e^{-\frac{\phi}{2}}}{1+\sqrt{1+\frac{\xi\Lambda}{3}\,e^{2\phi}}}\,, \tag{2.9}$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right]^7}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3}\,e^{2\phi}}\right)-13\,\Lambda\,e^{2\phi}\right]. \tag{2.10}$$

For:

$\xi = 1$

$\Lambda \simeq \frac{4\pi^2}{25}$

$\phi = 0.989117352243$

From

$$e^{2C} = \frac{2\xi e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3} e^{2\phi}}},$$

we obtain:

((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))))))

Input interpretation:

$$\frac{2 e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}}$$

Result:

0.382082347529...

0.382082347529….

Series representations:

$$\frac{2 e^{-0.98911735224300000/2}}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.98911735224300000}}{3\times 25}}} = 2 \Bigg/ \left(e^{0.4945586761215000}\left(1+\sqrt{\frac{4 e^{1.978234704486000}\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{2 e^{-0.98911735224300000/2}}{1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.98911735224300000}}{3\times 25}}} = 2 \Bigg/ \left(e^{0.4945586761215000}\left(1+\sqrt{\frac{4 e^{1.978234704486000}\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k \left(e^{1.978234704486000}\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{2\,e^{-0.9891173522430000/2}}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}} =$$

$$\frac{2}{e^{0.4945586761215000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

1+1/(((4((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))))))))

Input interpretation:

$$1+\frac{1}{4\times\frac{2\,e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}}}$$

Result:

1.65430921270...

1.6543092….. We note that, the result 1.6543092... is very near to the 14th root of the following Ramanujan’s class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Indeed:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4}$$
$$\times\left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}} = \left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. $\square$

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

Series representations:

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}} =$$
$$1+\frac{e^{0.49455867612150000}}{8}+\frac{1}{8}\,e^{0.49455867612150000}\sqrt{\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.97823470448600000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}} =$$
$$1+\frac{e^{0.49455867612150000}}{8}+\frac{1}{8}\,e^{0.49455867612150000}\sqrt{\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.97823470448600000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}=1+\frac{e^{0.4945586761215000}}{8}+$$

$$\frac{1}{8}\,e^{0.4945586761215000}\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

And from

$$\frac{h^2}{32}=\frac{e^{-4\phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3}e^{2\phi}}\right]^7}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3}e^{2\phi}}\right)-13\,\Lambda\,e^{2\phi}\right].$$

we obtain:

e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))-13*(4Pi^2)/25*e^(2*0.989117352243)]

Input interpretation:

$$\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\,(4\,\pi^2)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\,(4\,\pi^2)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\,(4\,\pi^2)\right)e^{2\times 0.989117352243}\right)\right)$$

Result:

$-0.034547055658\ldots$

-0.034547055658…

Series representations:

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times0.9891173522430000}}{3\times25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times0.9891173522430000}\right)\right.\right.$$
$$\left.\left. e^{-4\times0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)=$$
$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-\right.\right.\right.$$
$$25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}$$
$$\left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\right.$$
$$\left.\left.\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right.\right.$$
$$\left.e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7=$$
$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-\right.\right.\right.$$
$$25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}$$
$$\left.\left.\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\right.$$
$$\left.\left.\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(\left(42\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right.\right.$$
$$\left.e^{-4\times 0.9891173522430000}\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7=$$
$$-\left(\left(42\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\right.\right.\right.$$
$$\left.\left.\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(25\right.$$
$$e^{5.934704113458000}$$
$$\left.\left.\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

47 *1/(((-1/((((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))))))

Input interpretation:

$$47\left(-\left(1/1/\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)\right)$$

Result:

1.6237116159...

1.6237116159…. result that is an approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$-\left(47/1/\left(e^{-4\times 0.98911735224300000}\left(42\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}-\frac{1}{25}\left(4\pi^2\right)13\,e^{2\times 0.98911735224300000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{\left(4\pi^2\right)e^{2\times 0.98911735224300000}}{3\times 25}}\right)^7\right)=$$

$$\left(1974\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)$$

$$-\left(47\Big/1\Big/\left(e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)=$$

$$\left(1974\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)$$

$$-\left(47\Big/1\Big/\left(e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\,(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)=$$

$$\left(1974\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-25\,e^{1.978234704486000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(25\,e^{5.934704113458000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

And again:

32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))

Input interpretation:

$$32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)$$

Result:

−1.1055057810...

-1.1055057810….

We note that the result -1.1055057810…. is very near to the value of Cosmological Constant, less 10^{-52} , thence 1.1056, with minus sign

Series representations:

$$\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\;-\right.\right.\right.$$
$$\left.\left.\left.\frac{1}{25}\,(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/$$
$$\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$
$$-\left(\left(1344\left(-25\,e^{1.978234704486000}+52\,e^{3.956469408972000}\,\pi^2-\right.\right.\right.$$
$$25\,e^{1.978234704486000}\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}$$
$$\left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(25\,e^{5.934704113458000}\right.$$
$$\left.\left.\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\;\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)$$

$$\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Bigg/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(1344\left(-25\, e^{1.978234704486000}+52\, e^{3.956469408972000}\,\pi^2-25\, e^{1.978234704486000}\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Bigg/\left(25\, e^{5.934704113458000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)$$

$$\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Bigg/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7 =$$

$$-\left(\left(1344\left(-25\, e^{1.978234704486000}+52\, e^{3.956469408972000}\,\pi^2-25\, e^{1.978234704486000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Bigg/\left(25\, e^{5.934704113458000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

And:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))]))))]^5

Input interpretation:

$$-\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)^5$$

Result:

1.651220569...

1.651220569…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Series representations:

$$-\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5=$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)\Big/\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{35}\right)$$

$$-\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\middle/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)\middle/$$

$$\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{35}\right)$$

$$-\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\middle/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)^5 =$$

$$\left(4\,385\,270\,057\,140\,224\left(-25+52\, e^{1.978234704486000}\,\pi^2-25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^5\right)\middle/$$

$$\left(9\,765\,625\, e^{19.78234704486000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^{35}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

We obtain also:

-[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))])))))]^1/2

Input interpretation:

$$-\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)}$$

Result:

-0
$1.0514303501\ldots i$

Polar coordinates:

$r = 1.05143035007$ (radius), $\theta = -90°$ (angle)

1.05143035007

Series representations:

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\left(4\,\pi^2\right)13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(\left(1+\sqrt{1+\frac{\left(4\,\pi^2\right)e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)\right)}=-\frac{8}{5}\sqrt{21}\sqrt{\left(\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)\right)}$$

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)} = -\frac{8}{5}\sqrt{21}$$

$$\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)}$$

$$-\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\, 13\, e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\, e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)} =$$

$$-\frac{8}{5}\sqrt{21}\sqrt{\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\, e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

1 / -[32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))])))]^1/2

Input interpretation:

$$-\left(1\Bigg/\left(\sqrt{\left(32\left(\frac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}\right)^7}\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}\left(4\pi^2\right)\right)e^{2\times 0.989117352243}\right)\right)\right)\right)}\right)\right)$$

Result:

0.95108534763... i

Polar coordinates:

$r = 0.95108534763$ (radius), $\theta = 90°$ (angle)

0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index n_s, to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}} \approx 0.9568666373$$

Series representations:

$$-\left(1\Big/\left(\sqrt{\left(\left(32\, e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)}\right)\right) =$$

$$-\left(5\Big/\left(8\sqrt{21}\sqrt{\left(\left(\left(25-52\, e^{1.978234704486000}\,\pi^2+25\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\, e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}\right)^7\right)\right)\right)}\right)\right)$$

$$-\left(1\Bigg/\left(\sqrt{\left(\left(32\,e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}-\frac{1}{25}\,(4\,\pi^2)\,13\,e^{2\times0.9891173522430000}\right)\right)\right)\Bigg/\left(1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)}\right)\right)=$$

$$-\left(5\Bigg/\left(8\,\sqrt{21}\,\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7\right)\right)}\right)\right)$$

$$-\left(1\Big/\left(\sqrt{\left(\left(32\,e^{-4\times 0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}-\frac{1}{25}\right.\right.\right.\right.}\right.\right.$$

$$\left.\left.\left.(4\pi^2)\,13\,e^{2\times 0.9891173522430000}\right)\right)\right)\Big/$$

$$\left.\left.\left.\left(1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times 0.9891173522430000}}{3\times 25}}\right)^7\right)\right)\right) =$$

$$-\left(5\Big/\left(8\sqrt{21}\sqrt{\left(\left(25-52\,e^{1.978234704486000}\,\pi^2+25\sqrt{z_0}\right.\right.}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(e^{3.956469408972000}\right.$$

$$\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\right.$$

$$\left.\left.\left.\left.\left.\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)^7\right)\right)\right)\right)$$

for $(\text{not}\ (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

From the previous expression

$$\frac{\dfrac{e^{-4\times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}\right)^7}}{\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}}-13\left(\frac{1}{25}(4\pi^2)\right)e^{2\times 0.989117352243}\right)\right)}$$

= -0.034547055658…

we have also:

1+1/(((4((2*e^(-0.989117352243/2))) / ((((1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243)))))))))) + (-0.034547055658)

Input interpretation:

$$1+\frac{1}{4\times\frac{2\,e^{-0.989117352243/2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times 0.989117352243}}}}-0.034547055658$$

Result:

1.61976215705...

1.61976215705….. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}-0.0345470556580000=$$
$$0.9654529443420000+\frac{e^{0.49455867612150000}}{8}+\frac{1}{8}\,e^{0.49455867612150000}\sqrt{\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.97823470448600000}\,\pi^2\right)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2\times 0.98911735224300000}}{3\times 25}}}}-0.0345470556580000=$$
$$0.9654529443420000+\frac{e^{0.49455867612150000}}{8}+\frac{1}{8}\,e^{0.49455867612150000}\sqrt{\frac{4\,e^{1.97823470448600000}\,\pi^2}{75}}\sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.97823470448600000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{1}{\frac{4\left(2\,e^{-0.98911735224300000/2}\right)}{1+\sqrt{1+\frac{(4\pi^2)\,e^{2\times0.98911735224300000}}{3\times25}}}}-0.0345470556580000 =$$

$$0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+$$

$$\frac{1}{8}\,e^{0.4945586761215000}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000}\,\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2\times2\times8$

1024

$8^4 = 8^2\times2^6$

True

$8^4 = 4096$

$8^2\times2^6 = 4096$

$2^{13} = 2\times8^4$

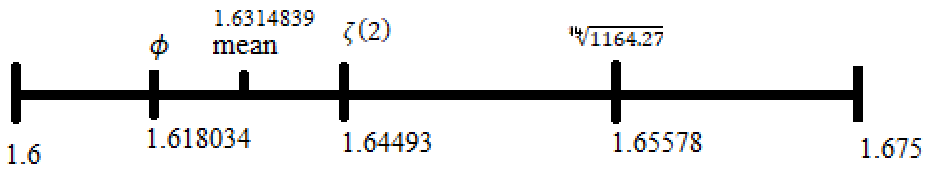
True

$2^{13} = 8192$

$2\times8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

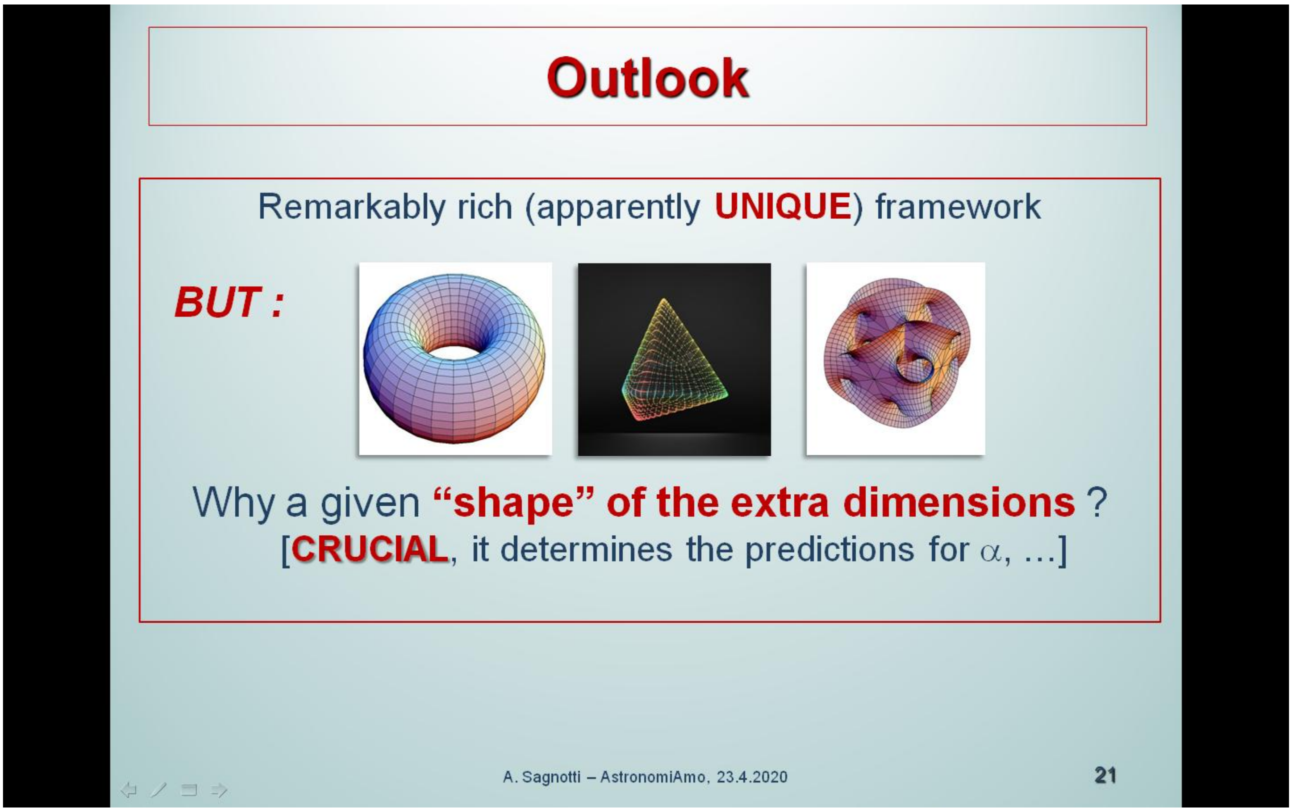
"Golden" Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

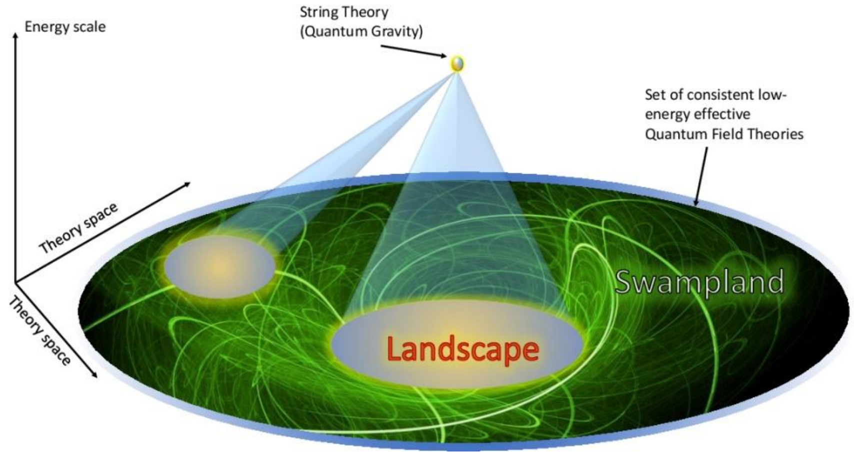


Fig. 1

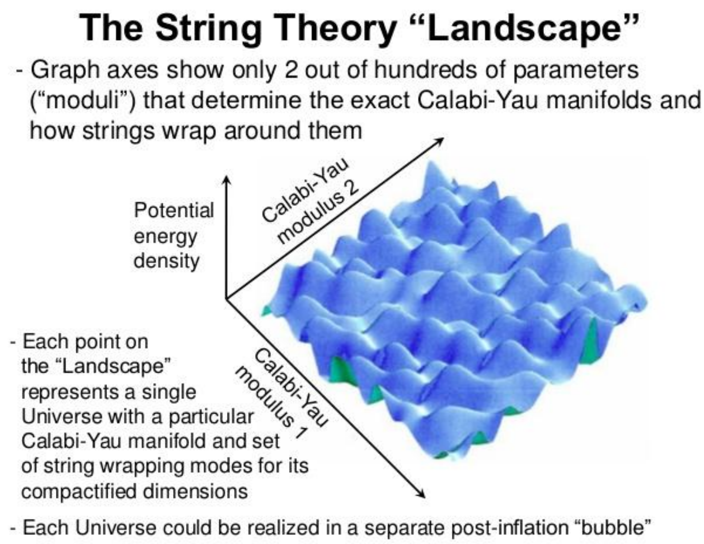


Fig. 2

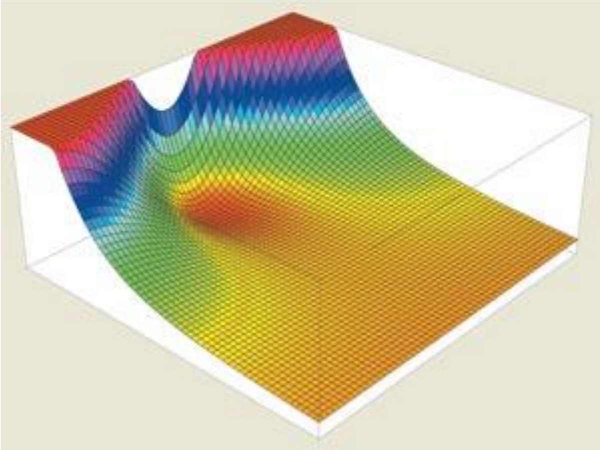


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

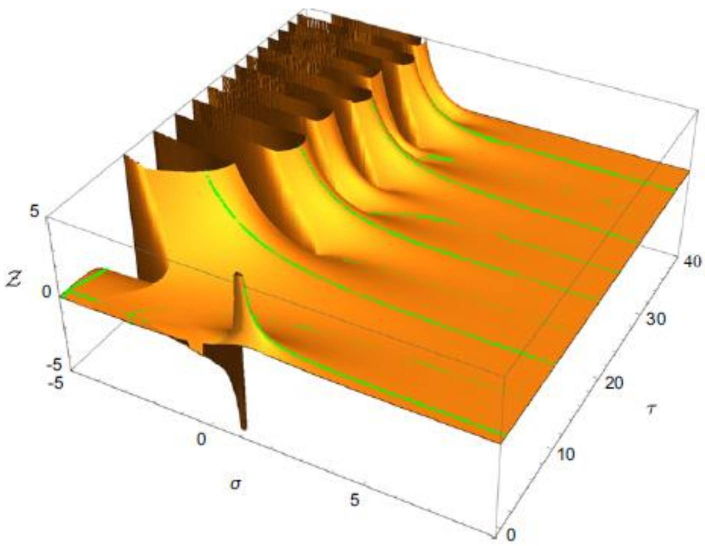


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \, \frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots.$

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
...

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4 i \pi^2}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293…

$1.6449340668\ldots. = \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

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